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Original Article

Minimum Quadratic Unbiased Estimators For Parameters Of First Order Moving Average Disturbances Regression Model With Box And Cox Transformation

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ABSTRACT

Box and Cox Transformation may be considered as general transformation which may contain several types of linear and non linear transformations as its special cases.This transformation its special cases.This transformation ha a wide number of applications in theory of statistics ,Applied Regression analysis and Time series Analysis.

In this Research Article, A new method of Estimation has been Suggested to estimate the parameters of the first order Moving Average regression Model with Box and cox Transformation for Dependent Variable.

Keywords:

Cox Transformation , Minimum Quadratic Unbiased Estimators , Parameters.

I.INTRODUCTION

The usual purpose of the transformation is to change the scale of the measurements in order to make the analysis more valid. The main purposes of transformations are:

i.to stabilize variances;
ii.to linearize relationships;
iii.to make the probability distributions as normal;
iv.to simplify the handling of data with other ackward features; and
v.to enable results to be presented in an acceptable scale of measurement.

Suitable transformations of data can frequently be found that will reduce a theoretically nonlinear model to a linear form. These transformations are said to be linearizable and comprise a class of functions that may either occur in practice or may themselves provide reasonable approximations to functions that occur in practice. Linearizing may require transforming both the independent and the dependent variables. An important class of linearizable functions is are the power or multiplicative models of the exponential form.

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Linear transformation may be useful in simplifying arithmetic computations. The logarithmic, square root and angular transformations may be considered as the nonlinear transformations in which equal increments on the original scale do not usually correspond to equal increments on the new scale.

In the present study,an estimation method has been suggested to estimate the parameters of First order Moving Average regression model with Box and Cox transformation for dependent Variable by using Internally Studentized residuals.

Ii. Estimation Of Parameters Of Linear Regression Model With In The Box And Cox Transformation:

Consider a linear regression model with Box and Cox transformation for dependent variable as,

$$Y_{nx1}^{(\lambda)} = X_{nxk}\beta_{kx1} + \in_{nx1} \qquad \text{.... (2.1)}$$

Where,

$Y^{(\lambda)}$ is a (nx1) vector of observations on dependent Variable with the Box and Cox transformation;

X is a (nxk) matrix of fixed observations on k independent variables;

β is a (kx1)vector of parameters;

and $\in$ is a (nx1) vector of random disturbances.

The transformed observations on dependent variable are given by

$$Y_i^{(\lambda)} = \left.\begin{matrix} \left[\dfrac{Y_i^{\lambda} - 1}{\lambda}\right], if \lambda \neq 0 \\ \ln(Y_i), \text{ if } \lambda = 0. \end{matrix}\right\} \qquad \text{.... (2.2)}$$

Assumptions about the model are given by,

(i) Rank of X is k < n

(ii) The elements of one of columns of X are all unity and

(iii) $\in_i \xrightarrow{iid} N(0, \sigma^2)$.

The logarithmic likelihood function is given by

$$\ln L(\lambda, \beta, \sigma^2; Y, X) = -\frac{n}{2}\ln(2\Pi\sigma^2) - \frac{1}{2\sigma^2}\left[Y^{(\lambda)} - X\beta\right]\left[Y^{(\lambda)} - X\beta\right] + \ln J. \qquad \text{.... (2.3)}$$

Where J is the Jacobian of the transformation on the dependent variable which is given by

$$J = \left|\frac{\partial Y_i^{(\lambda)}}{\partial Y_i}\right| = \prod_{i=1}^{n} Y_i^{\lambda-1} \qquad \text{.... (2.4)}$$

For a given λ, the maximum likelihood estimators for β and σ^2 are given by

$$\hat{\beta}(\lambda) = \left(X^{\prime} X\right)^{-1} X^{\prime} Y^{(\lambda)} \qquad \text{.... (2.5)}$$

And

$$\hat{\sigma}^2(\lambda) = \frac{\left[Y^{(\lambda)} - X\hat{\beta}(\lambda)\right]\left[Y^{(\lambda)} - X\hat{\beta}(\lambda)\right]}{n} \qquad \text{.... (2.6)}$$

Substituting (2.4), (2.5), (2.6) in to (2.3) gives the concentrated log likelihood function as

$$\ln L(\lambda; Y, X) = -\frac{n}{2}\left[\ln(2\Pi) + 1\right] - \frac{n}{2}\ln\hat{\sigma}^2(\lambda) + (\lambda - 1)\sum_{i=1}^{n}\ln(Y_i) \qquad \text{.... (2.7)}$$

Estimation of λ:

Step (1): An initial value for λ may be chosen from a selected range such as (-1, 1) or (-2, 2) or extend this range.

Step (2): For each chosen Value of λ say $\lambda^{\prime}$ the $Y_i^{(\lambda^{\prime})}$'s may be evaluated by

$$Y_i^{*} = Y_i^{(\lambda^{\prime})} = \left.\begin{matrix} \dfrac{Y_i^{\lambda^{\prime}} - 1}{\lambda^{\prime}}, if\ \lambda^{\prime} \neq 0, i = 1,2,\ldots\ldots, n \\ \ln Y_i, \text{if } \lambda^{\prime} = 0 \end{matrix}\right\} \qquad \text{.... (2.8)}$$

Step (3): Fit the linear regression model

$$Y_{nx1}^{*} = X_{nxk}\beta_{kx1} + \in_{nx1} \qquad \text{.... (2.9)}$$

and find the OLS residual vector as

$$e_{\lambda^{\prime}}^{*} = \left(Y^{*} - X\hat{\beta}\right) \qquad \text{.... (2.10)}$$

Define the Internally studentized residuals as

$$S_i^{(I)} = \frac{e^*_{\lambda^| i}}{\hat{\sigma}\sqrt{1-v_{ii}}} \qquad \text{.... (2.11)}$$

Where $\hat{\sigma}^2 = \frac{\sum_{i=1}^{n} e^{*^2}_{\lambda^| i}}{n-k}$ = the residual mean sum of squares

V= $(v_{ij}) = X(X^| X)^{-|} X^|$ such that

$$e^*_{\lambda^|} = (I-V)Y^* \qquad \text{.... (2.12)}$$

Also obtain the Internally studentized residual sum of squares as $\sum_{i=1}^{n} S_i^{(I)^2}$.

Step (4): Plot the points $\left(\lambda^|, \sum_{i=1}^{n} S_i^{(I)^2}\right)$, for different $\lambda^|$, on a graph and join them by means of a smoothed curve. Identify that value of $\lambda^|$ for which the lowest point of the curve lies. Let it be $\hat{\lambda}_1$. This $\hat{\lambda}_1$ gives the maximum likelihood estimator of λ .

The maximum likelihood estimators for β and σ^2 can be obtained from (2.5) and (2.6) as $\hat{\beta}(\lambda_1)$ and $\hat{\sigma}^2(\lambda_1)$ respectively.

ie $$\hat{\beta}(\hat{\lambda}_1) = (X^| X)^{-|} X^| Y^{(\hat{\lambda}_1)}$$

and

$$\hat{\sigma}^2(\hat{\lambda}_1) = \frac{\left[Y^{(\hat{\lambda}_1)} - X\hat{\beta}(\hat{\lambda}_1)\right]^| \left[Y^{(\hat{\lambda}_1)} - X\hat{\beta}(\hat{\lambda}_1)\right]}{n}$$

Iii. Estimation Of Parameters Of Linear Regression Model With The Disturbances In First Order Moving Average Regression Model:

Consider the linear regression model with first order moving average disturbances as

$$Y_t = X_t^| \beta + \in_t \text{, t =1, 2, ..., n} \qquad \text{.... (3.1)}$$

Such that $\in_t = u_t - \delta\, u_{t-1}$, t = 1, 2,.....,n

Where

Y_t is the t^{th} observation on the dependent variable,

X_t is a (kx1) vector containing the t^{th} observation on k non stochastic regressors;

β is (kx1) parametric vector;

$\in_t$ is the disturbance term following a first order moving average scheme with unknown parameter δ;

and $E(u_t) = 0, \forall$ t

$$E(u_t u_{t+s}) = \left.\begin{matrix} \sigma^2, \text{if s} = 0 \\ 0, \text{otherwise} \end{matrix}\right\}$$

One can write the model (3.1) in matrix notation as

$$Y_t = X_{nxk}\beta_{kx1} + \in_{nx1} \quad \text{.... (3.2)}$$

With $$E[\in] = 0 \text{ and } E[\in\in^{|}] = \sigma^2 V \quad \text{.... (3.3)}$$

Where

$$V = \begin{bmatrix} 1+\delta^2 & -\delta & 0 & . \; . \; . & 0 & 0 \\ -\delta & 1+\delta^2 & -\delta & . \; . \; . & 0 & 0 \\ 0 & -\delta & 1+\delta^2 & . \; . \; . & 0 & 0 \\ . & . & . & . \; . \; . & . & . \\ . & . & . & . \; . \; . & . & . \\ . & . & . & . \; . \; . & . & . \\ 0 & 0 & 0 & . \; . \; . & 1+\delta^2 & -\delta \\ 0 & 0 & 0 & . \; . \; . & -\delta & 1+\delta^2 \end{bmatrix}. \quad \text{.... (3.4)}$$

The OLS estimator of β is given by

$$\hat{\beta} = (X^{|}X)^{-1} X^{|}Y \quad \text{.... (3.5)}$$

This is unbiased with Covariance matrix.

$$E[\hat{\beta} - \beta][\hat{\beta} - \beta]^{|} = \sigma^2 (X^{|}X)^{-1} X^{|}VX (X^{|}X)^{-1} \quad \text{.... (3.6)}$$

The Generalized least squares estimator for β can be obtained by minimizing

$$(Y - X\beta)^{\prime} V^{-1} (Y - X\beta) = \in^{\prime} V^{-1} \in$$

and it gives the estimator as

$$\tilde{\beta} = \left(X^{\prime} V^{-1} X\right)^{-1} X^{\prime} V^{-1} Y \qquad \text{.... (3.7)}$$

This GLS estimator is unbiased with Covariance matrix

$$E\left[\tilde{\beta} - \beta\right]\left[\tilde{\beta} - \beta\right]^{\prime} = \sigma^2 \left(X^{\prime} V^{-1} X\right)^{-1}.$$

It is possible to derive a matrix P such as

$$P^{\prime} Y = P^{\prime} X\beta + P^{\prime} \in. \qquad \text{.... (3.8)}$$

A simpler alternative is to rewrite the model (3.1) or (3.2) as

$$Y^{*} = X^{*}\beta + Z^{*}\eta + u \qquad \text{.... (3.9)}$$

Where

$\eta = u_0$ and

$$Y_t^{*} = Y_t + \delta\, Y_{t-1}^{*}, \quad Y_0^{*} = 0$$

$$X_t^{*} = X_t + \delta\, X_{t-1}^{*}, \quad X_0^{*} = 0 \qquad \text{.... (3.10)}$$

$$Z_t^{*} = \delta\, Z_{t-1}^{*}, \quad Z_0^{*} = -1$$

The minimization of $u^{\prime} u$ with respect to δ, β and η is the same as the minimization of $\in^{\prime} V^{-1} \in$ with respect to δ and β.

Estimation of δ

Generally δ is unknown. It can be estimated by using sample autocorrelation coefficient, which is given by

$$\hat{\rho} = \frac{\sum_{t=2}^{n} e_t^{*} e_{t-1}^{*}}{\sum_{t=1}^{n} e_t^{*2}} \qquad \text{.... (3.11)}$$

Where e_t^{*} is the t[th]element of the Internallystudentized residual vector.

We have

$$e_t^* = \frac{e_t}{\hat{\sigma}\sqrt{1-v_{tt}}} \qquad \text{.... (3.12)}$$

Where e_t is the t^{th} element of the OLS residual vector

$$e = Y - X\hat{\beta} = MY = M \in, \qquad \mathrm{M} = \mathrm{I} - \mathrm{X}\left(\mathrm{X}^{|}X\right)^{-|}X^{|}$$

$$\hat{\sigma}^2 = \frac{\sum_{t=1}^{n} e_t^2}{n-k} \qquad \text{.... (3.13)}$$

$$V = \left(\left(v_{ij}\right)\right) = X\left(X^{|}X\right)^{|}X^{|}.$$

We have,

$$\rho = \frac{Cov\left(\in_t, \in_{t-1}\right)}{Var\left(\in_t\right)} = -\frac{\delta}{1+\delta^2}\left[\because \; from(5.6.3)\right] \qquad \text{.... (3.14)}$$

Substituting $\hat{\rho}$ for ρ in (3.14) yields

$$\hat{\rho} = -\frac{\delta}{1+\delta^2}$$

or

$$\hat{\rho}\,\delta^2 + \delta + \hat{\rho} = 0 \qquad \text{.... (3.15)}$$

Solving (3.15) for δ gives an estimator for δ as

$$\hat{\delta} = \frac{-1+\sqrt{1-4\hat{\rho}^2}}{2\hat{\rho}}, \quad \text{if } \left|\hat{\rho}\right| < 0.5 \qquad \text{.... (3.16)}$$

$$\hat{\delta} = -1 \quad \text{if } \hat{\rho} \geq 0.5 \qquad \text{..... (3.17)}$$

$$\hat{\delta} = 1 \quad \text{if } \hat{\rho} \leq \text{-}0.5 \qquad \text{..... (3.18)}$$

If X does not contained lagged dependent Variable, then $\hat{\rho}$ and $\hat{\delta}$ will be consistent estimates of ρ and δ respectively.

The estimator $\hat{\delta}$ is meaningful if $\left|\hat{\rho}\right| < 0.5.$ Since, $\hat{\rho}$ lies anywhere between -1 and 1, the condition $\left|\hat{\rho}\right| < 0.5$ is not small sample properties of $\hat{\delta}$ given in (3.16).

An alternative estimator for δ has been proposed according to a method suggested by ullah, Vinod and Singh (1986).

Rewrite the matrix V in (3.4) as

$$V = (1+\delta^2)I - \delta\,G = (1+\delta^2)(I+\rho\,G), \quad \rho = -\frac{\delta}{1+\delta^2} \qquad \text{.... (3.19)}$$

Where

$$G_{nxn} = \begin{bmatrix} 0 & 1 & 0 & . & . & . & 0 & 0 \\ 1 & 0 & 1 & . & . & . & 0 & 0 \\ 0 & 1 & 0 & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 0 & 1 \\ 0 & 0 & 0 & . & . & . & 1 & 0 \end{bmatrix} \qquad \text{.... (3.20)}$$

Now, one can write

$$E[ee^{\prime}] = \left.\begin{matrix} \sigma^2(MVM) \\ \sigma_{\in}^2(M+\rho MGM), \; \sigma_{\in}^2 = \sigma^2(1+\delta^2) \end{matrix}\right\} \qquad \text{.... (3.21)}$$

By collecting the $(t, t+1)^{th}$, t = 1, 2, ..., n-1, off diagonal elements on both sides of (3.21) and writing them in a column vector, one can obtain

$$E\left[\frac{e_*}{\sigma_{\in}^2}\right] = Z_1 + \rho\,Z_2 \qquad \text{.... (3.22)}$$

Or

$$\frac{e_*}{\sigma_{\in}^2} - Z_1 = \rho\,Z_2 + \gamma \qquad \text{.... (3.23)}$$

Where γ is a $(n-1)\times 1$ vector such that

$E(\gamma) = 0$ and

$$e_* = (e_{12}, e_{23}, ..., e_{n-1,n})^{\prime} \qquad \text{.... (3.24)}$$

Here, $e_{t\,t+1} = e_t e_{t+1}$

$Z_1 = (m_{12}, m_{22}, ..., m_{n-1,n})^{\prime}$

$$Z_2 = \left(Z_{12}, Z_{23}, \ldots\ldots\ldots, Z_{n-1,n}\right)^{|}$$

and, $Z_{t,t+1}$, $m_{t,t+1}$

and $e_{t\,t+1}$ are respectively

the $(t, t+1)^{th}$ element of MGM, M and $ee^{|}$.

One can estimate ρ unbiasedly by fitting equation (3.23) and it is given by

$$\tilde{\rho} = \left(Z_2^{|} Z_2\right)^{-|} Z_2^{|} \left(\frac{e_*}{\sigma_\epsilon^2} - Z_1\right) \qquad \ldots\ldots (3.25)$$

This estimator is like C.R.Rao's Minimum Quadratic unbiased estimator (MINQUE).

Generally, $\tilde{\sigma}_\epsilon^2$ can be replaced with its consistent estimator

$\tilde{\sigma}_\epsilon^2 = \dfrac{e^{|}e}{n}$ and rewrite the equation (3.23) as

$$Z = \rho\, Z_2 + \gamma, \qquad \ldots\ldots (3.26)$$

Where $Z = \left(\dfrac{e_*}{\tilde{\sigma}_\epsilon^2} - Z_1\right)$ and $\rho = -\dfrac{\delta}{1+\delta^2}$

Now, the MINQUE for ρ is given by

$$\tilde{\rho} = \left(Z_2^{|} Z_2\right)^{-|} Z_2^{|} Z. \qquad \ldots\ldots (3.27)$$

It can shown that $\tilde{\rho}$ is asymptotically unbiased estimator. Thus an estimator for δ can be obtained from (3.16) as

$$\tilde{\delta} = \frac{-1+\sqrt{1-4\tilde{\rho}^2}}{2\tilde{\rho}}, \qquad \text{if} \quad |\tilde{\rho}| < 0.5 \qquad \ldots\ldots (3.28)$$

Now, $\tilde{\delta}$ can be used in (3.4), (3.9) and (3.10) and hence obtain an estimator for β as $\tilde{\beta}^*$.

A nonlinear least squares estimator for δ say $\tilde{\tilde{\delta}}$ can be obtained by minimizing

$$\gamma^{|}\gamma = \left[Z - g(\delta) Z_2\right]^{|}\left[Z - g(\delta) Z_2\right] \qquad \ldots\ldots (3.29)$$

With respect to δ .

Where $$g(\delta) = -\frac{\delta}{1+\delta^2}$$

Iv. Estimation Of Parameters Of The First Order Moving Average Regression Model With Box And Cox Transformation:

Consider this Box and Cox first order moving average disturbances regression model with transformation for dependent variable as

$$Y_{nx1}^{(\lambda)} = X_{nxk}\beta_{kx1} + \in_{nxk} \qquad \text{.... (4.1)}$$

or $$Y_t^{(\lambda)} = X_t^{|}\beta + \in_t, \qquad t = 1, 2, \ldots\ldots, n; \qquad \text{.... (4.2)}$$

and $$\in_t = u_t - \delta\, \mathrm{u}_{t-1}, \qquad \text{.... (4.3)}$$

such that $$E[\in] = 0 \text{ and } E\left[\in\in^{|}\right] = \sigma^2 V$$

Where

$$V = \begin{bmatrix} 1+\delta^2 & -\delta & 0 & . & . & . & 0 & 0 \\ -\delta & 1+\delta^2 & -\delta & . & . & . & 0 & 0 \\ 0 & -\delta & 1+\delta^2 & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 1+\delta^2 & -\delta \\ 0 & 0 & 0 & . & . & . & -\delta & 1+\delta^2 \end{bmatrix} \qquad \text{.... (4.4)}$$

The logarithmic likelihood function is given by

$$\ln L(\lambda, \delta, \beta, \sigma^2; Y, X) = -\frac{n}{2}\ln(2\Pi\sigma^2) + \frac{1}{2}\ln\left(1 - \frac{\delta^2}{(1+\delta^2)^2}\right) - \frac{1}{2\sigma^2}\left(Y^{(\lambda)} - X\beta\right)V^{-1}\left(Y^{(\lambda)} - X\beta\right) + \ln J. \qquad \text{.... (4.5)}$$

Where J is the Jacobian of the transformation on the dependent variable, which is given by

$$J = \left|\frac{\partial Y_t^{(\lambda)}}{\partial Y_t}\right| = \prod_{t=1}^{n} Y_t^{\lambda-1} \qquad \text{.... (4.6)}$$

Maximizing (4.5) with respect to β and σ^2 given both $\lambda = \hat{\lambda}_1$ and $\delta = \tilde{\tilde{\delta}}$, one can obtain the Maximum likelihood estimators for β and σ^2 as

$$\hat{\beta}\left(\hat{\lambda}_1, \tilde{\tilde{\delta}}\right) = \left[X' \tilde{\tilde{V}}^{-1} X\right]^{-1} \left[X' \tilde{\tilde{V}}^{-1} Y^{(\hat{\lambda}_1)}\right] \qquad .... (4.7)$$

And

$$\hat{\sigma}^2\left(\hat{\lambda}_1, \tilde{\tilde{\delta}}\right) = \frac{\left[Y^{(\hat{\lambda}_1)} - X\hat{\beta}\left(\hat{\lambda}_1, \tilde{\tilde{\delta}}\right)\right]' (\tilde{\bar{V}}) \left[Y^{(\hat{\lambda}_1)} - X\hat{\beta}\left(\hat{\lambda}_1, \tilde{\tilde{\delta}}\right)\right]}{n} \qquad .... (4.8)$$

respectively.

Here
$$\tilde{\tilde{V}} = \begin{bmatrix} 1+\tilde{\tilde{\delta}}^2 & -\tilde{\tilde{\delta}} & 0 & . & . & . & 0 & 0 \\ -\tilde{\tilde{\delta}} & 1+\tilde{\tilde{\delta}}^2 & -\tilde{\tilde{\delta}} & . & . & . & 0 & 0 \\ 0 & -\tilde{\tilde{\delta}} & 1+\tilde{\tilde{\delta}}^2 & . & . & . & 0 & 0 \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ . & . & . & . & . & . & . & . \\ 0 & 0 & 0 & . & . & . & 1+\tilde{\tilde{\delta}}^2 & -\tilde{\tilde{\delta}} \\ 0 & 0 & 0 & . & . & . & -\tilde{\tilde{\delta}} & 1+\tilde{\tilde{\delta}}^2 \end{bmatrix} \qquad .... (4.9)$$

This estimator $\tilde{\tilde{\delta}}$ can be used in (3.4), (3.9) and (3.10) and hence obtain an estimator for β as $\tilde{\beta}^*$.

It can be shown that $\tilde{\tilde{\rho}} = -\dfrac{\tilde{\tilde{\delta}}}{1+\tilde{\tilde{\delta}}^2}$ will always lies between -0.5 and 0.5.

V. Conclusions:

In the present research article,the linear regression model with box and cox transformation has been first specified and the parameters have been estimated by using the method of the Maximum likelihood estimation, secondly,the linear regression model with the first order moving average disturbances has been specified and its parameters have been estimated by using internally studentized residuals.Finally by combining these two specifications a new estimation method suggested for the Box and Cox first order moving average disturbances regression model with transformation for dependent variable.

Bibliography

1. Baltagi, B.H. (1999), "Econometrics", Second Revised Edition, Springer – Verlag Berlin Heidelberg New York.
2. Bhuyan, K.C. (2005), "Multivariate Analysis and its Applications", New central Book Agency (p) Ltd, Kolkata.
3. Cochrane, D., and Orcutt, G.H. (1949), "Applications of least squares Regression to Relationships containing Autocorrelated Error Terms", Journal of American Statistical Association, 44, 32 – 61.
4. Goldfeld, S.M., and Quandt, R.E. (1972), "Non Linear Methods in Econometrics", Amsterdam: North Holland Publishing Co.
5. Halawa, Adel M. (1996), "Estimating the Box-Cox Transformation Via an Artificial Regression Model", Communications in Statisitcs, - Simulation and Computation, 25(2), 331 – 350.
6. Judge, G.G., Griffiths, W.E., Hill, R.C., Lutkepohl, H., Lee, T.S. (1985), "The Theory and Practice of Econometrics", Second Edition, John Wiley and Sons, New York.
7. Kadiyala, K.R. (1968), "A Transformation used to circumvent the problem of autocorrelation", Econometricia, 36, 93 – 96.
8. Mccullagh, P., Nelder, J.A.(1989), "Generlized Linear Models", second Edition London, Chapman and Hall, New York, London.
9. Oxley, L.T. (1986), "Box-Cox Power transformations and the demand money functions in France, Germany, Italy and the Netherlands", Journal of Applied Statistics, 13, 67 – 75.
10. Seaks, T.G., Vines, D.P. (1990), " A Monte Carlo Evaluation of the Box- Cox difference Transformation", Review of Economics and Statistics, 72, 506 -515.
11. Ullah, A., Vinod, H.D., Singh, R.S. (1986), "Estimation of Linear Models with Moving Average Disturbances", Journal of Quantitative Economics, 2, 137 – 152.