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Original Article

Testing Linear Restrictions In Linear Statistical Models

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ABSTRACT

Criteria for Testing linear restrictions on parameters in linear statistical models with the heteroscedastic errors have been developed in the present study. A simple test procedure has been suggested to test the linear restrictions in the linear regression models after making a pre-test for the error variance. Some existing tests for general linear hypotheses have been modified with reference to the linear statistical model under the problem of hetroscedasticity.

Keywords:

Testing Linear Restrictions , Linear Statistical Models , heteroscedastic errors .

I.INTRODUCTION

In applied econometric work, researchers are often interested in testing the various forms of hypotheses about the parameters of the linear statistical model. If the parameters of the linear model obey the certain set of linear restrictions, then one may wish to reestimate the linear model, incorporating the restrictions in the estimation process. One important reason for such reestimation is that it will improve the efficiency of the estimates of the parameters of the linear model. For instance, if the hypothesis of constant returns to scale is not rejected for a Cobb-Douglas production function, the reestimation process would yield the production function with estimated elasticities which sum to unity.

The increasing demand for restricted estimation in applied econometrics, coupled with greater use of many parameter systems, necessiates a reexamination of traditional estimation techniques with respect to precision in computation. A scheme for efficient numerical estimates and testing of stochastic linear systems in which subsets of parameters may be constrained by linear restrictions.

Since, the early period of 1960's there has been a considerable growth in the research about the testing linear restrictions on parameters of the linear statistical model.

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Recently much research work has been done by developing procedures for estimating the linear regression models subject to linear parametric equality restrictions, linear parametric inequality restrictions, or a mixture of the two.

The problem of testing linear restrictions on parameters in two or more linear regression models arises with considerable frequency in econometrics. In the present study, some modified methods for testing linear restrictions in the linear statistical models have been proposed in the presence of heteroscedastic disturbances.

The three classical test procedures in econometrics namely Wald, Likelihood Ratio and Lagrange Multiplier tests have been modified and used for testing linear restrictions in generalized linear regression models.

The main contributors regarding the problem of testing linear restrictions on parameters of linear statistical models are : Atiqullah (1969), Wallace (1972), McElroy (1977), King and Smith (1986), Ohtani (1987, 1993), Farebrother (1988), Firoozi (1993), Kiefer, Vogelsang and Bunzel (2000), Ravikumar, Ray and Savin (2000) and others.

II. TESTING LINEAR RESTRICTIONS IN GENERALIZED LINEAR REGRESSION MODELS AFTER A PRE-TEST FOR DISTURBANCE VARIANCE

Consider a generalized linear regression model with nonspherical disturbances as

$$Y_{nx1} = X_{nxk}\,\beta_{kx1} + \epsilon_{nx1}\ ,\ \epsilon \sim N\,(0,\ \sigma^2\,\psi) \qquad \ldots (2.1)$$

Where Y is an (nx1) vector of observations on the dependent variable;

X is an (nxk) matrix of observations on independent variables of rank k;

β is (kx1) vector of unknown regression coefficients;

ϵ is an (nx1) vector of normal disturbances with mean zero and positive definite covariance matrix ψ.

Suppose that one may have the auxiliary regression sample (Y_A, X_A) in addition to the regression sample (Y,X) at hand. The auxiliary generalised linear regression model is written as

$$Y_A = X_A\,\beta_A + \epsilon_A\ ,\ \epsilon_A \sim N\,(0,\ \sigma_A^2\ \psi_A) \qquad \ldots (2.2)$$

Where

Y_A is an (mx1) vector;

X_A is (mxp) matrix of rank p;

β_A is a (px1) vector of regression coefficients;

ϵ_A is an (mx1) vector of normal disturbances with mean zero and positive definite covariance matrix ψ_A.

It is assumed that the regression sample at hand (Y,X) and the auxiliary regression sample (Y_A, X_A) are mutually independent.

Consider the general linear hypothesis of q linear restrictions on regression coefficients as

$H_0 : R\,\beta = r \quad \sim H_1 : R\,\beta \neq r.$

Where R and r are (qxk) and (qx1) matrices with known elements and the rank of R is q.

The unrestricted EGLS estimators for β and β_A are given by

$$\hat{\tilde{\beta}} = (X' \hat{\psi}^{-1} X)^{-1} (X' \hat{\psi}^{-1} Y) \qquad \text{... (2.3)}$$

$$\hat{\tilde{\beta}}_A = \left(X'_A \hat{\psi}_A^{-1} X_A\right)^{-1} \left(X'_A \hat{\psi}_A^{-1} Y_A\right) \qquad \text{... (2.4)}$$

Where $\hat{\psi}$ and $\hat{\psi}_A$ are the estimates of ψ and ψ_A respectively.

Also, the unbiased estimators of σ^2 and σ_A^2 are given by

$$\tilde{\sigma}^2 = \frac{\left(Y - X\hat{\tilde{\beta}}\right)' \hat{\psi}^{-1} \left(Y - X\hat{\tilde{\beta}}\right)}{(n-k)} \qquad \text{... (2.5)}$$

$$\tilde{\sigma}_A^2 = \frac{\left(Y - X\hat{\tilde{\beta}}_A\right)' \hat{\psi}_A^{-1} \left(Y - X\hat{\tilde{\beta}}_A\right)}{(m-p)} \qquad \text{... (2.6)}$$

one may use the prior information that $\sigma^2 = \sigma_A^2$ but $\beta \neq \beta_A$, and the pooled estimator of σ^2 as

$$\sigma^{*2} = [(n-k)\, \tilde{\sigma}^2 + (m-p)\, \hat{\sigma}_A^2] \,/\, (n-k+m-p) \qquad \text{... (2.7)}$$

If one wishes to test the linear restrictions on regression coefficients $H_0 : R\,\beta = r \sim H_1 : R\,\beta \neq r$, the proposed test statistic is given by

$$\tilde{F} = \frac{\left(R\hat{\tilde{\beta}} - r\right)' \left[R\left(X'\hat{\psi}^{-1}X)^{-1}\right)^{-1} R'\right]^{-1} \left(R\hat{\tilde{\beta}} - r\right) \Big/ q}{\tilde{\sigma}^2} \qquad \text{... (2.8)}$$

It well known that $\tilde{F}$ is distributed as the noncentral F distribution with degrees of freedom q and (n–k) and noncentrality parameter

$$\tilde{\lambda} = (R\beta - r)' [R(X'\psi^{-1}X)^{-1}R']^{-1} (R\beta - r) / \sigma^2 \qquad \text{... (2.9)}$$

If one has the prior information that $\sigma^2 = \sigma_A^2$ but $\beta \neq \beta_A$, the proposed alternative test statistic for $H_0 : R\beta = r$, by using the pooled estimator of σ^2 as

$$\tilde{F}^* = \frac{\left(R\hat{\tilde{\beta}} - r\right)' \left[R\left(X'\hat{\psi}^{-1}X\right)^{-1}R'\right]^{-1} \left(R\hat{\tilde{\beta}} - r\right) \Big/ q}{\sigma^{*2}} \sim \text{Noncentral}$$

$$F\left[q, (n-k-m-p), \tilde{\lambda}\right] \qquad \text{... (2.10)}$$

In the test statistic $\tilde{F}^*$ the never – pool estimator $\tilde{\sigma}^2$ in $\tilde{F}$ is replaced by the pooled estimator σ^{*2}. Since the degrees of freedom of σ^{*2} are greater than the degrees of freedom of $\tilde{\sigma}^2$, the test given by $\tilde{F}^*$ has a higher power than the test given by $\tilde{F}$.

If $\sigma^2 = \sigma_A^2$ is uncertain, one may conduct a pre-test for the hypothesis $H_0^* : \sigma^2 = \sigma_A^2$ and the test statistic is given by

$$F_0 = \frac{\tilde{\sigma}^2}{\tilde{\sigma}_A^2} \sim F_{[(n-k),\,(m-p)]} \; [\because \tilde{\sigma}^2 \text{ and } \tilde{\sigma}_A^2 \text{ are independent}] \qquad \text{... (2.11)}$$

The pooled estimator σ^{*2} is used if H_0^* is accepted in the pre-test and the never – pool estimator $\tilde{\sigma}^2$ is used if H_0^* is rejected. The resultant estimator σ_2^* is known as the pre-test estimator of the disturbance variance.

If H_0^* is accepted in the pre-test then one may use $\tilde{F}^*$; and if H_0^* is rejected in the pre-test then one may be use $\tilde{F}$ for testing $H_0 : R\beta = r$.

Thus, the proposed test procedure gives a two-stage generalized least squares test which consists of the pre-test for H_0^* : $\sigma^2 = \sigma_A^2$ followed by the main test for general linear hypothesis $H_0 : R\beta = r$.

According to Ohtani (1987) if the alternative in the pre-test is two-sided (i.e., H_1^* : $\sigma^2 \neq \sigma_A^2$) then the two- stage generalized least squares test may be formulated as follows :

(i) If $F_L < F_0(\text{Cal}) < F_U$, then reject H_0 if $\tilde{F}_{Cal}^* \geq F_{Cri}^*$ but accept H_0 if $\tilde{F}^* < F_{Cri}^*$;

(ii) If $F_0(\text{Cal}) \leq F_L$ or $F_0(\text{Cal}) \geq F_U$, then reject H_0 if $\tilde{F}_{Cal} \geq F_{Cri}$ but accept H_0 if $\tilde{F}_{Cal} \geq F_{Cri}$.

Where F_L and F_U are lower and upper critical values in the pre-test and F_{Cri} and F_{Cri}^* are critical values in the main tests.

If one may have the prior information that $\sigma^2 \geq \sigma_A^2$, the alternative in the pre-test is one-sided (i.e., H_1^* : $\sigma^2 \geq \sigma_A^2$), then the test procedure is given by putting $F_L = 0$ in the above formulation.

III. MODIFIED WALD, LIKELIHOOD RATIO AND LAGRANGE MULTIPLIER TESTS FOR TESTING LINEAR RESTRICTIONS IN GENERALIZED LINEAR REGRESSION MODELS.

Consider the linear regression model

$$Y_{nx1} = X_{nxk}\,\beta_{kx1} + \in_{nx1} \qquad \ldots (3.1)$$

With $\in \sim N(0, \sigma^2\,\Omega)$

Where Y is an (nx1) vector of observations on the dependent variable;

X is an (nxk) matrix of observations on the nonstochastic explanatory variables;

β is a (kx1) vector of regression coefficients;

$\in$ is an (nx1) error vector which follows a multivariate normal distribution with zero mean vector and dispersion matrix $\sigma^2\,\Omega$;

Ω is a positive definite matrix and there exists a nonsingular matrix P such that

$$P\,P^{l} = \Omega$$

or

$$(P^{l}\,\Omega^{-l}P)^{-l} = I$$

Pre-multiplying (3.1) by P^{-1} on both sides, one may get

$$P^{-1}Y = P^{-1} X \beta + P^{-1} \in$$

or $$Y^* = X^* \beta + \in^* \qquad \ldots (3.2)$$

Where $Y^* = P^{-1}Y$, $X^* = P^{-1}X$ and $\in^* = P^{-1} \in$.

Further, $E(\in^*) = 0$ and $E[\in^* \in^{*'}] = P^{-1}[\in \in'] \; P^{-1'} = \sigma^2 P^{-1}\Omega \; P^{-1'}$

$$\Rightarrow E[\in^* \in^{*'}] = \sigma^2 (P^{-1}\Omega^{-1} P)^{-1} = \sigma^2 I_n.$$

Now, the BLUE for β in (3.2) is the Generalized Least Squares (GLS) estimator which is given by

$$\tilde{\beta} = (X^{*'}X^*)^{-1} X^{*'}Y^* = (X\Omega^{-1}X)^{-1} X\Omega^{-1}Y \qquad \ldots (3.3)$$

with $$\text{Var}(\tilde{\beta}) = \sigma^2 (X^{*'}X^*)^{-1} = \sigma^2 (X\Omega^{-1}X)^{-1} \qquad \ldots (3.4)$$

If one defines $E[\in \in'] = \sigma^2 \Omega = \phi$ then

$$\tilde{\beta} = (X'\phi^{-1}X)^{-1} X'\phi^{-1}Y$$

with $$\text{Var}(\tilde{\beta}) = (X'\phi^{-1}X)^{-1}$$

An unbiased estimator for σ^2 is $\tilde{\sigma}^2$ which is given by

$$\tilde{\sigma}^2 = \frac{e^{*'} e^*}{n-k} = \frac{\left(Y^* - X^* \tilde{\beta}\right)'\left(Y^* - X^* \tilde{\beta}\right)}{n-k}$$

$$= \frac{\left(Y - X\tilde{\beta}\right)\left(p^{-1'} p^{-1}\right)\left(Y - X\tilde{\beta}\right)}{n-k}$$

$$\Rightarrow \qquad \tilde{\sigma}^2 = \frac{\left(Y - X\tilde{\beta}\right)' \Omega^{-1}\left(Y - X\tilde{\beta}\right)}{n-k} \qquad \ldots (3.5)$$

Suppose the disturbances are heteroscedastic but not serially correlated, then

$$\Omega = \text{diag}\{\sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2\} \qquad \ldots (3.6)$$

In this case, one may have

$$P = \text{diag}\{\sigma_1, \sigma_2, \ldots, \sigma_n\},\ P^{-1} = \text{diag}\ \left\{\frac{1}{\sigma_1}, \frac{1}{\sigma_2}, \ldots, \frac{1}{\sigma_n}\right\} = \Omega^{-1/2}$$

and $\quad \Omega^{-1} = \text{diag}\left\{\frac{1}{\sigma_1^2}, \frac{1}{\sigma_2^2}, \ldots, \frac{1}{\sigma_n^2}\right\}$.

Pre-multiplying the regression model by $\Omega^{-1/2}$ is equivalent to dividing the i^th^ observation of the model by σ_i. This yields the new disturbance $\left(\frac{\in_i}{\sigma_i}\right)$ have zero mean and homoscedastic variance σ^2. Thus, the regression runs $Y_i^* = \left(\frac{Y_i}{\sigma_i}\right)$ on $X_{ij}^* = \left(\frac{X_{ij}}{\sigma_i}\right)$

for i = 1, 2, …, n;
and j = 1, 2, …, k.

Consider $\in^* = P^{-1}\in = \Omega^{-1/2}\in$, where $\in \sim N(0, \sigma^2\,\Omega)$.

Then, $\in^* \sim N(0, \sigma^2 I_n)$. The likelihood function of $\in^*$ is given by

$$L(\in^* / \beta, \sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} \exp\{-\in^{*\prime}\in^*/2\sigma^2\} \qquad \ldots (3.7)$$

By making the transformation $\in = P\in^* = \Omega^{-1/2}\in^*$, one may get

$$L\ (\in|\beta,\sigma^2) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} |\Omega^{-1/2}|\ \exp\{-\in^{\prime}\Omega^{1/2}\in / 2\sigma^2\}$$

... (3.8)

Where $|\Omega^{-1/2}|$ is the jacobian of the inverse transformation.

By substituting $\in = Y - X\beta$, one may obtain the likelihood function as

$$L(Y/\ \beta,\ \sigma^2\Omega) = \left(\frac{1}{2\pi\sigma^2}\right)^{n/2} |\Omega^{-1/2}|\ \exp\ \{-(Y-X\beta)^{\prime}\ \Omega^{-1}(Y-X\beta)\ /\ 2\sigma^2\}$$

... (3.9)

Maximizing (3.9) with respect to β is equivalent to minimizing $\in^{*\prime}\in^{*}$ with respect to β. This yields that the OLS estimator on the transformed model will be the maximum likelihood estimator. Thus, the GLS estimator $\tilde{\beta}$ is the maximum likelihood estimator of β.

Also, the maximum likelihood estimator of σ^2 is given

by $$\tilde{\sigma}^2 = \frac{(Y - X\tilde{\beta})^{\prime}\ \Omega^{-1}(Y - X\tilde{\beta})}{n-k}$$ ... (3.10)

or $$\hat{\sigma}^2 = \frac{(n-k)\tilde{\sigma}^2}{n}$$ ... (3.11)

In fact, one may have, $\tilde{\beta} \sim N\ [\ \beta,\ \sigma^2\ (X\Omega^{-1}X)^{-1}]$

and $$\frac{(n-k)\tilde{\sigma}^2}{\sigma^2} \sim \chi^2_{(n-k)}.$$

Consider the general linear hypotheses consists of q linear restrictions about β as

$H_0 :\ R\beta = r$

Where R is (qxk) matrix of known coefficients

R is (qx1) vector of known constants

The test statistic for testing H_0, using the transformed model (3.2) is given by

$$\chi^2 = (R\,\tilde{\beta} - r)' \,[R\,(X^{*\prime}X^{*})^{-1}\,R']^{-1}(R\,\tilde{\beta} - r) \,/\, \sigma^2 \sim \chi^2_q \qquad \ldots (3.12)$$

$$\Rightarrow \chi^2 = (R\,\tilde{\beta} - r)' \,[R\,(X'\Omega^{-1}X)^{-1}\,R']^{-1}(R\,\tilde{\beta} - r) \,/\, \sigma^2 \sim \chi^2_q \qquad \ldots (3.13)$$

Further, one can obtain the restricted GLS estimator for β

as $$\tilde{\beta}_{RLS} = \tilde{\beta}\ (X'\Omega^{-1}X)^{-1}\,R'[\,R(X'\Omega^{-1}X)^{-1}\,R']^{-1}\,(R\tilde{\beta} - r) \qquad \ldots (3.14)$$

Write $\phi = \sigma^2\Omega$ and consider the general linear model (3.1) with $\in\ \sim N(0, \phi)$. By substituting $\sigma^2\,\Omega = \phi$ in the likelihood function and maximizing it w.r.t. β and φ without imposing the H_0, one may obtain the unrestricted maximum likelihood estimators of β and φ are

$$\tilde{\beta}_{UR} = (X'\hat{\phi}^{-1}X)^{-1}\,(X'\hat{\phi}^{-1}Y) \qquad \ldots (3.15)$$

and $\hat{\phi}$ respectively.

The conditional maximum likelihood estimators of β and φ under the restriction H_0, can be similarly obtained as

$$\tilde{\beta}_{R} = (X'\tilde{\phi}^{-1}X)^{-1}\,(X'\tilde{\phi}^{-1}Y) - (X'\tilde{\phi}^{-1}X)^{-1}\,R'[R\,(X'\tilde{\phi}^{-1}X)^{-1}R']^{-1}$$

$$[R\,(X'\tilde{\phi}\,X)^{-1}\,(X'\tilde{\phi}^{-1}Y) - r\,] \qquad \ldots (3.16)$$

or $$\tilde{\beta}_{R} = \hat{\beta}_{UR} - (X'\tilde{\phi}^{-1}X)^{-1}\,R'[R\,(X'\tilde{\phi}^{-1}X)^{-1}R']^{-1}\,(R\,\hat{\beta}_{UR} - \Omega\,] \qquad \ldots (3.17)$$

and $\tilde{\phi}$ respectively.

Let $\hat{\beta}_{UR}$ be the unrestricted maximum livelihood estimator of β conditional on the restricted variance – covariance estimator $\tilde{\phi}$ and let $\hat{\beta}_{R}$ be the restricted maximum likelihood estimator of β under H_0 conditional on the unrestricted variance – covariance estimator $\hat{\phi}$, then one may have

$$\hat{\beta}_{UR} = (X'\tilde{\phi}\,X)^{-1}\,X'\tilde{\phi}^{-1}Y \qquad \ldots (3.18)$$

and $\hat{\beta}_R = \tilde{\beta}_{UR} - (X^{\prime} \hat{\phi}^{-1} X)^{-1} R^{\prime} [R (X^{\prime} \hat{\phi}^{-1} X)^{-1} R^{\prime}]^{-1} (R \tilde{\beta}_{UR} - r)$

... (3.19)

If heteroscedasticity exists and $\in$ is free from autocorrelation then ϕ can be expressed as a diagonal matrix,

i.e., $\phi = \text{diag}\{ \sigma_1^2, \sigma_2^2, \ldots, \sigma_n^2 \}$ ... (3.20)

It is proposed to estimate the elements of ϕ by using the GLS residuals, namely

$$\tilde{\phi}_{GLS} = \text{diag}(\tilde{e}_1^2, \tilde{e}_2^2, \ldots, \tilde{e}_n^2) \quad \ldots (3.21)$$

Where $\tilde{e}_i^2 = Y_i - X_i^{\prime} \tilde{\beta}$, i = 1, 2, ..., n.

Here, X_i is a (kx1) vector of observations on the non-stochastic regressors;

$\tilde{\beta} = (X^{\prime} \tilde{\phi}_{GLS}^{-1} X)^{-1} X^{\prime} \tilde{\phi}_{GLS}^{-1} Y$ is the GLS estimator of β.

For testing $H_0 : R\beta = r \sim H_1 : R\beta \neq r$

the modified likelihood ratio [LR] criterion is given by

$$(LR)^* = -2 \log_e \Lambda^* \quad \ldots (3.22)$$

Where

$$\Lambda^* = \frac{\underset{R\beta=r}{Max} L(Y/\beta,\phi)}{Max L(Y/\beta,\phi)} = \frac{\text{Conditional maximum likelihood function under } H_0}{\text{Unconditional maximum likelihood function}}$$

$$\frac{L(Y/\tilde{\beta}_R, \tilde{\phi})}{L(Y/\tilde{\beta}_{UR}, \hat{\phi})}$$

one may obtain the modified likelihood ratio criterion as

$$(LR)^* = -2 \log \left\{ \frac{L(Y / \tilde{\beta}_R, \tilde{\phi})}{L(Y / \tilde{\beta}_{UR}, \hat{\phi})} \right\} = \left| \tilde{e}^{\prime} \tilde{\phi}^{-\prime} \tilde{e} \right| - \left| \hat{e} \hat{\phi}^{-\prime} \hat{e} \right| \qquad \ldots (3.23)$$

where

$$\hat{e} = [Y - X\ \tilde{\beta}_{UR}] \text{ and } \tilde{e} = [Y - X\ \tilde{\beta}_R]$$

If both the estimators of β are conditional on a restricted variance– covariance estimator $\tilde{\phi}$, then the modified likelihood ratio criterion based on $\tilde{\phi}$ is given by

$$(LR)^{**} = -2 \log \left\{ \frac{L(Y / \tilde{\beta}_R, \tilde{\phi})}{L(Y / \hat{\beta}_{UR}, \tilde{\phi})} \right\} = \left| \tilde{e}^{\prime} \tilde{\phi}^{-\prime} \tilde{e} \right| - \left| e^{*\prime} \tilde{\phi}^{-\prime} e^{*} \right| \qquad \ldots (3.24)$$

Where $\tilde{e}$ = [Y – X $\tilde{\beta}_R$] and e* = [Y – X $\tilde{\beta}_{UR}$]

If both the estimators of β are conditional on the unrestricted variance – covariance estimator $\hat{\phi}$, then the modified likelihood ratio criterion based on $\hat{\phi}$ is given by

$$(LR)^{***} = -2 \log \left\{ \frac{L(Y / \tilde{\beta}_R, \hat{\phi})}{L(Y / \hat{\beta}_{UR}, \hat{\phi})} \right\}$$

$$= (e^{**\prime}\ \hat{\phi}^{-\prime}\ e^{**}) - [\hat{e}^{\prime}\ \hat{\phi}^{-\prime}\ \hat{e}] \qquad \ldots (3.25)$$

Where e**= (Y – X $\hat{\beta}_R$) and $\hat{e}$ = [Y – X $\tilde{\beta}_{UR}$]

Consider R $\tilde{\beta}_{UR}$ ~ N [Rβ, R(X$^{\prime}$ $\hat{\phi}^{-\prime}$X)$^{-\prime}$ R$^{\prime}$].

Now, the Wald test statistic for testing H_0 is given by

$$W = (R\ \tilde{\beta}_{UR} - r)^{\prime}\ [\ R(X^{\prime}\ \hat{\phi}^{-\prime} X)^{-\prime}\ R^{\prime}]^{-\prime}\ (R\ \tilde{\beta}_{UR} - r) \qquad \ldots (3.26)$$

Using (3.19), it can be easily shown that

$e^{**} = Y - X\,\hat{\beta}_R$; $\hat{e} = Y - X\,\tilde{\beta}_{UR}$ and hence

$$e^{**} = \hat{e} + X\,(X^{I}\hat{\phi}^{-I}X)^{-I}\,R^{I}[\,R(X^{I}\hat{\phi}^{-I}X)^{-I}\,R^{I}]^{-I}\,(R\,\tilde{\beta}_{UR} - r) \qquad \dots (3.27)$$

and $$e^{**I}\hat{\phi}^{-I}e^{**} = [\hat{e}^{I}\hat{\phi}^{-I}\hat{e}] + (R\,\tilde{\beta}_{UR} - r)\,[\,R(X^{I}\hat{\phi}^{-I}X)^{-I}\,R^{I}]^{-I}\,(R\,\tilde{\beta}_{UR} - r) \qquad \dots (3.28)$$

The cross product terms are zero because $X^{I}\hat{\phi}^{-I}\hat{e} = 0$.

∴ The modified Wald test statistic is given by

$$W^{*} = [\,e^{**I}\hat{\phi}^{-I}e^{**}] = [\hat{e}^{I}\hat{\phi}^{-I}\hat{e}\,] = -\,2\log\left\{\frac{L(Y/\tilde{\beta}_R,\hat{\phi})}{L(Y/\hat{\beta}_{UR},\hat{\phi})}\right\} = (LR)^{***} \qquad \dots (3.29)$$

Thus, the modified Wald test statistic can be interpreted as a modified Likelihood Ratio criterion conditional on $\hat{\phi}$, the unrestricted maximum likelihood estimator of ϕ.

Similarly, the Lagrange Multiplier (LM) test statistic for testing

H_0 : $[\,R(X^{I}\tilde{\phi}^{-I}X)^{-I}\,R^{I}]^{-I}\,(R\,\hat{\beta}_{UR} - r) = 0$ is given by

$$(LM) = (R\,\hat{\beta}_{UR} - r)^{I}\,[\,R(X^{I}\tilde{\phi}^{-I}X)^{-I}\,R^{I}]^{-I}\,(R\,\hat{\beta}_{UR} - r) \qquad \dots (3.30)$$

using (3.16), it can be easily shown that

$$\hat{e} = e^{*} + X(X^{I}\tilde{\phi}^{-I}X)^{-I}\,R^{I}[\,R(X^{I}\tilde{\phi}^{-I}X)^{-I}\,R^{I}]^{-I}\,(R\,\hat{\beta}_{UR} - r) \qquad \dots (3.31)$$

Where $\tilde{e} = (Y - X\tilde{\beta}_R)$ and $e^{*} = (Y - X\tilde{\beta}_{UR})$ and hence,

$$[\tilde{e}^{I}\tilde{\phi}^{-I}\tilde{e}\,] = [e^{*I}\tilde{\phi}^{-I}e^{*}] + (R\,\hat{\beta}_{UR} - r)^{I}\,[\,R(X^{I}\tilde{\phi}^{-I}X)^{-I}\,R^{I}]^{-I}\,(R\,\hat{\beta}_{UR} - r) \qquad \dots (3.32)$$

The cross product terms are zero because $X^{I}\tilde{\phi}^{-I}e^{*} = 0$

∴ The modified Lagrange Multiplier test statistic is given by

$$(LM)^* = [\tilde{e}^{\prime}\, \tilde{\phi}^{-1}\, \tilde{e}\,] - [e^{*\prime}\, \tilde{\phi}^{-1} e^*] = -2 \log \left\{ \frac{L(Y/\tilde{\beta}_R, \tilde{\phi})}{L(Y/\hat{\beta}_{UR}, \tilde{\phi})} \right\} = (LR)^{**} \qquad \dots (3.33)$$

Thus, the modified Lagrange Multiplier test statistic be interpreted as a modified Likelihood Ratio Criterion conditional on $\tilde{\phi}$, the restricted maximum likelihood estimator of ϕ.

It can be shown that

$$W^* \geq (LR)^* \geq (LM)^* \qquad \dots (3.34)$$

VI. CONCLUSIONS

In the present research work, an attempt has been made by developing some criteria for testing linear restrictions on parameters in linear regression models with the heteroscedastic disturbances. Further , the famous test procedures namely Wald, Likelihood Ratio and Lagrange Multiplier tests for testing linear restrictions have been modified under the problem of heteroscedasticity.

A simple test procedure has been suggested to test the linear restrictions in the generalized linear regression models after making a pre-test for the error variance. This test is to be used when auxiliary regression sample is available in addition to the regression sample at hand. The proposed test uses the pre-test pooled estimator for the disturbance variance. Thus, the proposed test procedure gives a two-state generalized least squares test which consists of the pre-test for disturbance variance followed by the main test for the general linear hypothesis about the parameters of the generalized linear regression model.

Further, the basic three tests namely the Wald, the Likelihood Ratio and the Lagrange Multiplier tests based on restricted least squares estimators have been modified and used for testing the general linear hypothesis consists of a set of linear restrictions about the parameters of a linear regression model. The modified lagrange multiplier statistic and the modified Wald test statistics have been interpreted as the modified likelihood ratio test statistics conditional on the restricted and unrestricted maximum likelihood estimators of the error variance – covariance matrix respectively.

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