PRIMARY ARTICLE

An Operational Calculus Method In Spherical Co-ordinate For Some Mixed Boundary Problems

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ABSTRACT

The present paper devoted to dual integral equations involving non stationary Heat conduction equation in the Laplace Transform for two dimensional symmetrical under mixed discontinuous boundary conditions acted on level surface of semi space in spherical co-ordinates we find temperature distribution function of moving solid object along a surface of semi space with velocity v by consideration of non-stationary heat conduction equation and Heat source $m_1(r, t)$ inside a disc of radius c, r < c, outside of disc r > c, a temperature function $m_2(r, t)$ is given. We use operational calculus method and dual integral equation for the solution of given boundary value problem.

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Dual Integral equation, Spherical co-ordinate, Mixed boundary conditions, Heat conduction equation, Temperature function.

INTRODUCTION :

In a study of mixed boundary value problems in mathematical physics we come across dual integral equation method to solve the equation of elliptic type in different areas of applications such as: diffraction, potential theory, elasticity and steady state heat equation [6-7]. Dual integral equation method has been developed and applied quite successfully in various physical and engineering problems. There are several papers available involving non-stationary heat equation with application of dual integral were published [1, 3, 4]. In this paper we are interested to find the solution of heat equation related to moving solid heat object with the use of operational calculus method and dual integral equation. We take a Bessi function of the first kind of order zero and unknown function, weight function, free term with dependence of a Laplace transform parameter. By using some known discontinuous integrals, the dual integral equations were reduced to a Fredholm integral equation of second kind.

MATHEMATICAL FORMULATION OF THE PROBLEM:

Let us consider a process of heat body moves along x-axis having velocity v on a semi-space z > 0 in a Cartesian co-ordinates xyz of a solid object, with a heat source moves inside a disk $x^2 + y^2 < c^2$, outside of disk $x^2 + y^2 < c^2$, a temperature functions is given. We have to find the temperature distribution of this body.

Now It is required to solve a non-stationary heat conductivity differential equation

$$\nabla^2 T - \frac{v\,\partial x\,T}{a\ \partial} = \frac{1}{a}\frac{\partial T}{\partial \tau}\ , \ v > 0,\ \ \tau > 0, a > 0 \qquad (2.1)$$

Where, ∇^2 is Laplacian Operator
T = T (x, y, z) x, y, z Cartesian co-ordinates

A is heat coefficient and is independent of temperature or co-ordinates.Now, use the substitution $T = u\,.\,\exp(-w\,x)$ where $w = \frac{v}{2a}$, equation (2.1) should be written as

$$\nabla^2 u - w^2 u = \frac{1}{a}\frac{\partial u}{\partial \tau} \qquad (2.2)$$

u = u (x, y, z)

$\nabla^2 u$ in spherical co-ordinates should be written as

$$\nabla^2 u = [\frac{\partial^2}{\partial \phi^2} + \frac{1}{r}\frac{\partial}{\partial r}\left(r \frac{\partial}{\partial r}\right)]\ u$$

The initial condition is

U (r,Φ, 0) = T (r, Φ, 0) – T_0 = 0 (2.3)

The boundary conditions are,

$$\left(\frac{\partial u}{\partial r}\right)_{r=0} = \left(\frac{\partial u}{\partial r}\right)_{r\to\infty} = (u)_{\phi\to\infty} = \left(\frac{\partial u}{\partial \phi}\right)_{r\to\infty}$$

=0 (2.4)

and under the mixed discontinuous bounded conditions along the level surface Φ = 0.

$$\left(\frac{\partial u}{\partial \phi}\right) = -m_1(r,\tau); \qquad r \in (0,c) \qquad (2.5)$$

$$u = -m_2(r,\tau); \qquad r \in (0,\infty) \qquad (2.6)$$

where, $m_1(r,\tau)$ is heat source obey.

Newton's law of heating inside the disc r < c, $m_2(r, \tau)$ is a temperature function acted outside of disc r > c. The known functions $m_2(r, \tau)$, i = 1, 2 is continuous and having limited variations w.r.t. each of variables r and τ. More ever [4].

Due to these restrictions we apply Laplace transform with respect to t and Hankle transform with respect to r. Here we assume that the functions $m_1(r, \tau)$; i = 1, 2 have absolutely, continuous derivative with respect to r.

We may use Laplace transform from (2.2) to (2.6) such that

$$u(r, \phi, s) = \int_0^\infty u(r, z, \tau)\exp(-s\,\tau)\,dt$$

A general solution of given problem may be taken as

$$u(r, z, s) = \int_0^\infty G(p,s)\exp\{-\phi r(ps) J_0(pr)\,dp\}$$

(2.7)

where

$$r(p, s) = \sqrt{p^2 + (w^2 + s/a)}$$

(p,s) is unknown function to be determined.

To find unknown function A(p, s) we use the mixed boundary conditions (2.5) and (2.6) in the Laplace transform image, we get the pair of dual integral equation as

$$\int_0^\infty G(p,s) r(p,s) J_0(pr)\,dp = m_1(r,s)$$

$$r \in (0,c) \qquad (2.8)$$

Now we solve (2.7) and (2.8). For this purpose assume function $m_2(r, s)$ as

$$m_2(r, s) = \int_0^\infty H(y,s) J_0(pr)\,dp$$

(2.10)

where

H(p, s) is a known function.

Now, we apply the inverse Hankel transform to (2.9), so that

$$H(p, s) = \int_0^\infty y\,p\,m_2\,J_0(py)\,dy$$

Let us suppose an unknown function L(p, s) s.t. L (p, s) – H (p, s) = G (p, s). The unknown function L (p, s) is determined by the dual integral equation

$$_0 \quad L(p,s) r(p,s) J_0(pr)\,dp \quad M(r,s)$$

$$r \quad (0,c) \qquad (2.11)$$

$$_0 \quad L(p,s)\ J_0(pr)\,dp \quad 0$$

$$r \quad (0, \quad) \qquad (2.12)$$

where M(r, s) = m2(r, s) –

$$_0 \quad \sqrt{p^2 \quad d}\ H(p,s) J_0(pr)\,dp$$

$d = w^2 + s/a$

Now we use the relation

$$L(p, s) = \ _0 \quad (t,s)\ \sqrt{(t,p)}\ dt \qquad (2.13)$$

$$\sqrt{(t,p)} = \frac{p}{\sqrt{p^2 + d}} \sin\left\{t\sqrt{p^2 + d}\right\}$$

To solve the equations (2.10) and (2.11). We make use the discontinuous integrals [5]

$$\int_0^\infty \frac{p J_0(pr)}{\sqrt{p^2+d}} \sin\left(t\sqrt{p^2+d}\right) dp = \begin{cases} 0 & r > t \\ \dfrac{\cos\sqrt{(t^2-r^2)\,d}}{\sqrt{t^2-r^2}} & t > r \end{cases} \tag{2.14}$$

$$\int_0^\infty J_1(pr) \sin\left(t\sqrt{p^2+d}\right) dp$$

$$= \frac{1}{r} \begin{cases} \sin t\sqrt{d} - \dfrac{-t}{\sqrt{t^2-r^2}} \sin\left(\sqrt{t^2-r^2}\right) d & 0 < r < t < c \\ \sin t\sqrt{d} - \dfrac{-t}{\sqrt{t^2-r^2}} \exp\left(-\sqrt{r^2-t^2}\right) d & 0 < r < t < c \end{cases} \tag{2.15}$$

Substitute (2.12) into (2.11) and use (2.13). Also put (2.12) into (2.10) and use discontinuous integrals (2.15), we get a first kind singular Integral equation as

$$\int_0^r \frac{\sqrt{(r^2-t^2)\,d}}{\sqrt{r^2-t^2}} \lambda(t, s)\, dt$$

$$= M(r,s) + \int_0^c \left\{ \frac{\sin\sqrt{(t^2-r^2)\,d}}{\sqrt{t^2-r^2}} - \frac{\sin\sqrt{dt}}{t} \right\} \lambda(t,s)\, dt \quad r \in (0,c) \tag{2.16}$$

where $\lambda(t, s)$ is a unknown function to be determined and $\lambda(t, s) = t\,\mu(t, s)$. Now, we trend the equation (2.16) as an Abels integral equation a Fredholm integral equation of IInd Kind in obtained with unknown function $\lambda(t, s)$

$$\lambda(t,s) + \int_0^R \lambda(\xi,s)\, K(\xi,s,t)\, dt = F(t,s) \tag{2.17}$$

The free term and Kernel is given by

$$F(t,s) = \frac{2}{\pi}\frac{d}{dt}\int_0^t \frac{\sqrt{(t^2-y^2)\,d}}{\sqrt{(t^2-y^2)}} M(y,s)\, dy \tag{2.18}$$

$$K(t, \xi\ s) = \frac{2}{\pi}\frac{d}{dt}\int_0^t \frac{\cos\sqrt{(t^2-y^2)d}}{\sqrt{(t^2-y^2)}}\left\{\frac{\sin\sqrt{(\xi^2-y^2)d}}{\sqrt{\xi^2-y^2}} - \frac{\sin(\sqrt{d}\,\xi)}{\xi}\right\}dy \qquad (2.19)$$

We may solve Integral equation (2.11) by successive approximation technique expanding sinx and cosx in appropriate Maclaurins series. Finally we can use the expression

$$\lambda(t, s) = \sum_{m=-\infty}^{-1} \lambda_m(t)\, s^{m/2} + \exp(-ck) \sum_{m=0}^{\infty} \lambda_m(t)\, s^{\frac{m}{2}-1}, \quad k^2 = \frac{s}{a}$$

This theory can be used to solve many problems of Mathematical physics which involving the Heat conduction equation. Hence we obtain the similar results as boundary conditions acted on level surface of semi space in cylindrical co-ordinates.

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