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Primary Article

Patterns Of Neighbours For Block Design Constructed Using MOLS

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ABSTRACT

In certain situations of experimentation using usual blocking each experimental plot contains direct effect of the treatment applied to it together with effects of treatments percolating from its neighbouring left and right plots. This paper is concerned with the study of block designs of OS1 Series for two sided (left and right) second- order neighbour effects. Neighbour Designs can be constructed using MOLS for the parameters $v=s^2$, $b=s(s+1)$, $r=s+1$, $k=s$, $\lambda=1$ where s is either a prime number or power of a prime number. Here left and right second-order neighbours of every treatment are finding out and it is observed that these neighbours follow the property of circularity of second-order.

Keywords:

MOLS, BIBD, Border Plots, Neighbour Design, Second- order left and right neighbours, Circularity.

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1 Introduction:

In many experiments especially in agriculture, the response on a given plot may be affected by treatments on neighbouring plots as well as by the treatments applied to that plot. To diminish these undesirable effects, designs with neighbour balance properties are generally used. For this Rees (1967) introduced the concept and name them "neighbour designs". A design is said to be first - order neighbour – balanced if each treatment has every other treatment as a neighbour an equal number of times. A design is said to be second- order neighbour- balanced if each treatment has every other treatment as one plot away neighbour an equal number of times. Wilkinson et al. (1983) defined partially neighbour balanced if each experimental treatment has each other treatment as a neighbour, on either side, at most once. Bailey (2003) considered the one – sided neighbour effects only. Jaggi, Gupta and Ashraf (2006) suggested general method of construction of complete block designs which are partially balanced for neighbouring competition effects. A series of incomplete block designs partially balanced for neighbour effects has also been suggested. Iqbal et al. (2006) gave the construction of second-order neighbour designs using the method of Cyclic Shifts. Laxmi and Rani (2009) studied the designs of two sided (left and right) first order neighbour effects considering border plots. Neighbour Design constructed by the use of MOLS for the series $v=s^2$, $b=s(s+1)$, $r=s+1$, $k=s$, $\lambda=1$(where s is either a prime number or power of a prime number) and the patterns of neighbour treatments for every treatment were observed for the neighbour designs.

2 Neighbour Designs using BIBD:

In 1936 Yates introduced the concept of orthogonal series for B.I.B.D. with parameters $v=s^2$, $b=s(s+1)$, $r=s+1$, $k=s$, $\lambda=1$ and $v=b=s^2+s+1$, $r=k=s+1$, $\lambda=1$.The first series was named as OS1 series and the second series was named as OS2. For the construction of Neighbour Designs of OS1 series one may refer to Laxmi and Rani (2009). Here it is assumed that no treatment is (i) adjacent to itself and (ii) adjacent to any other treatment more than once.

2.1 Neighbour Design Using OS1 series when s=3.

The resultant neighbour design for OS1 series using MOLS for s=3 with parameters v=9, b=12, r=4, k=3, λ=1 is given as:

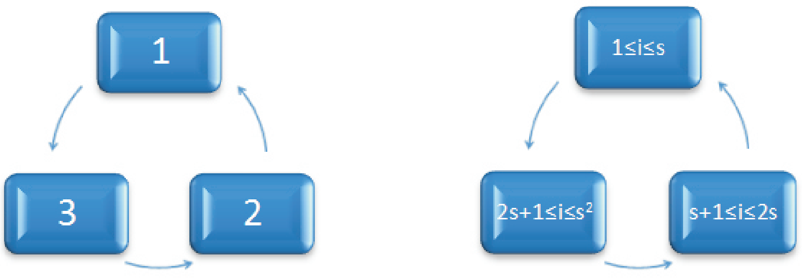
3	6	9	7	8	9	8	9	7	9	7	8
1	4	7	1	2	3	1	2	3	1	2	3
2	5	8	4	5	6	6	4	5	5	6	4
3	6	9	7	8	9	8	9	7	9	7	8
1	4	7	1	2	3	1	2	3	1	2	3

For OS1 series when s=3 the second- order left and right neighbours cannot be find out simultaneously (for getting second- order two sided neighbour treatment simultaneously there must be at least s$\geq$5.i.e.the block size k$\geq$5) that is only one directional second- order neighbours can be obtained for s=3. In the above design in block 1 treatment number 2 is the second- order left neighbour of treatment number 1. Similarly, all the second- order left neighbours can be obtained for treatment number 1 in which block the treatment number 1 appears. Thus a list of second-order left neighbours for treatment number 1 so obtained is 2, 4, 6 & 5. Further in block 1 treatment 3 is the second-order left neighbour of treatment number 2.In block 5 treatment number 5 is the second-order left neighbour of treatment number 2. Similarly all the other second -order left neighbours for treatment number 2 can be obtained in which block this treatment number appears. Thus a list of second-order left neighbours for treatment number 2 so obtained is 3, 5, 4 & 6. Similarly, all the second-order left neighbours for every other treatment can be obtained and these neighbours are shown in Table 1.

Table 1

I	Second-order left neighbours	Common neighbours	Other neighbour	Series
1	2,4,6,5	4,5,6	2	1≤i≤s
2	3,5,4,6	4,5,6	3	i.e.
3	1,6,5,4	4,5,6	1	1≤i≤3
4	5,7,9,8	7,8,9	5	s+1≤i≤2s
5	6,8,7,9	7,8,9	6	i.e.
6	4,9,8,7	7,8,9	4	4≤i≤6
7	8,1,3,2	1,2,3	8	2s+1≤i≤s^2
8	9,2,1,3	1,2,3	9	i.e.
9	7,3,2,1	1,2,3	7	7≤i≤9

From the Table 1 it is observed that treatment number 1 has second-order left neighbours are 2, 4, 6 & 5. As the treatment number 1 lies in the series (1$\leq$i$\leq$s) so the second-order common left neighbour series of it shall be (s+1$\leq$i$\leq$2s) which are there as s+1,s+2,s+3(2s). As the immediate second-order left neighbour of treatment number i=1 should be i-2. One more neighbour of treatment number 1 is treatment number 2 which simply can't be defined as i-2 immediate second-order left neighbour. As the property of circularity holds not only for the complete design but it also holds for each set of s treatments so the other member of treatment number 1 is 2. Now, the treatment number 2 has 3, 5, 4 & 6 as second-order left neighbours. As the treatment number 2 lies in series (1$\leq$i$\leq$s) which has (s+1$\leq$i$\leq$2s) as second-order left neighbour series. Immediate second-order left neighbour of treatment number 2 is 3 which simply can't be defined as i-2. In case of OS1 series , v=s^2 where s is a prime number or power of a prime number circularity not only holds for the complete design but it also holds for each set of s treatments. Hence for the treatment number 2 immediate second-order left neighbour is treatment number 3. For treatment number 3 immediate second-order left neighbour is simply i-2 i.e. treatment number 1 and common second-order neighbours are again 4 ,5, 6. The left circularity can be picturised as:



Left circularity

Secondly, it is observed that treatment number 4 has neighbours as 5,7,9&8. As the treatment number 4 lies in the series ($s+1\leq i\leq 2s$) so the second –order left series of it shall be ($2s+1\leq i\leq s^2$) which are there as 2s+1,2s+2,2s+3(s^2). One more second-order left neighbour of treatment number 4 is treatment number 5 which again can't be defined as i-2. As circularity not only holds for the complete design but it also holds for each set of s treatments. Hence for the treatment number 4 immediate second-order left neighbour is treatment number 5. Similarly, for treatment number 5 immediate second-order left neighbour is treatment number 6 because of the property of circularity and common second-order left neighbour series is ($2s+1\leq i\leq s^2$) i.e.7,8 and 9. Further for treatment number 6 the common second-order left neighbour series is ($2s+1\leq i\leq s^2$) and the other member of neighbour treatments is simply written as i-2 that is treatment number 4.

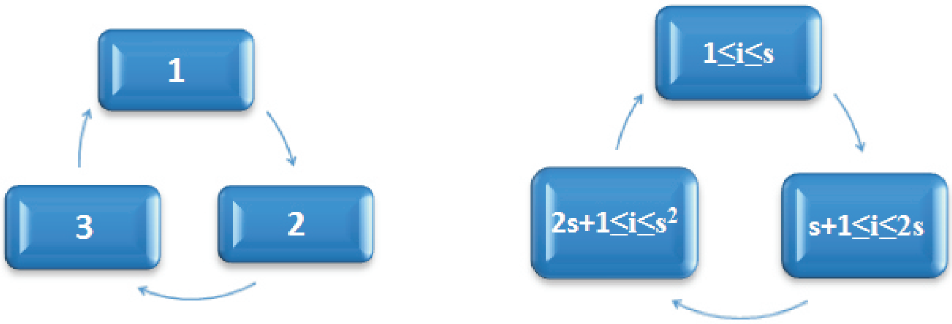
lastly, for treatment number 7, 8 & 9 the common second- order left neighbours are 1, 2 & 3. As the treatment number 7, 8 & 9 lies in the series ($2s+1\leq i\leq s^2$) so the common second-order left series of these should be ($1\leq i\leq s$) (as the circularity holds for the complete design as well as for each set of s treatments) which are there as 1,2,3 . The other member of treatment number 7 is 8 which is immediate second-order left neighbour because of the property of circularity. One more neighbour of treatment number 8 is 9 which simply can't be defined as i-2. As the property of circularity holds for each set of s treatments so treatment number 8 has immediate second-order left neighbour as treatment number 9. Similarly, for treatment number 9 immediate second-order left neighbour i-2 is treatment number 7.

Now, from the resulted neighbour design for OS1 series when s=3, the second-order right neighbours can be obtained. In the above design in block 1 treatment number 3 is the second- order right neighbour of treatment number 1. Similarly, all the second- order right neighbours can be obtained for treatment number 1 in which block the treatment number 1 appears. Thus a list of second-order right neighbours for treatment number 1 so obtained is 3,7,8&9. Further in block 1 treatment 1 is the second- order right neighbour of treatment number 2.In block 5 treatment number 8 is the second- order right neighbour of treatment number 2. Similarly, all the second -order right neighbours for treatment number 2 can be obtained in which block the treatment number 2 appears. Thus a list of second-order right neighbours for treatment number 2 so obtained is 1,8,7&9. Similarly, all the second-order right neighbours for every other treatment can be obtained and the resulted second-order right neighbours are shown in Table 2.

Table 2

i	Second-order right neighbours	Common neighbours	Other neighbour	Series
1	3,7,8,9	7,8,9	3	$1 \le i \le s$
2	1,8,7,9	7,8,9	1	i.e.
3	2,9,7,8	7,8,9	2	$1 \le i \le 3$
4	6,1,2,3	1,2,3	6	$s+1 \le i \le 2s$
5	4,2,3,1	1,2,3	4	i.e.
6	5,3,1,2	1,2,3	5	$4 \le i \le 6$
7	9,4,5,6	4,5,6	9	$2s+1 \le i \le s^2$
8	7,5,6,4	4,5,6	7	i.e.
9	8,6,4,5	4,5,6	8	$7 \le i \le 9$

From the Table 2 it is observed that treatment number 1 has neighbours 3,7,8&9 as second-order right neighbours. As the treatment number 1 lies in the series ($1 \le i \le s$) so the common second-order right neighbour series of it shall be ($2s+1 \le i \le 3s$) which are there as 2s+1,2s+2,2s+3(s^2) i.e.7,8,9. As the immediate second-order right neighbour of treatment number i=1 should be i+2 so the other member of neighbours is treatment number 3. Now, the treatment number 2 has 7,8,9,&1 as second-order right neighbours. As the treatment number 2 lies in the series ($1 \le i \le s$) and again ($2s+1 \le i \le 3s$) is the series of common second-order right neighbours. Immediate second-order right neighbour of treatment number 2 is 1 which simply can't be defined as i+2. In case of OS1 series, $v=s^2$ (where s is either a prime number or power of a prime number) circularity not only holds for the complete design but it holds for each set of s treatments. Hence for the treatment number 2 immediate second-order right neighbour is treatment number 1. As the property of circularity holds for each set of s treatments so treatment number 2 is the immediate second-order right neighbour of treatment number 3 and common second-order right neighbour are again 7, 8&9. The right circularity for can be picturised as:



Right circularity

Secondly, it is observed that treatment number 4 has second-order right neighbours are 6,1,2&3. As the treatment number 4 lies in the series ($s+1 \le i \le 2s$) so the common second –order right neighbour series of it shall be ($1 \le i \le s$) which are there as 1,2,3. As the immediate second-order right neighbour of treatment number i=4 should be i+2 so the other member of neighbours is treatment number 6. Further, the treatment number 5 has second-order right neighbours are 4,2,3,&1. As the treatment number 5 lies in the series ($s+1 \leq i \leq 2s$) which has ($1 \leq i \leq s$) as the series of common second-order right neighbours. One more second-order right neighbour of treatment number 5 is treatment number 4 which again can't be defined as i+2. As circularity not only holds for the complete design but it also holds for each set of s treatments, hence for the treatment number 5 immediate second-order right neighbour is treatment number 4. Similarly, for treatment number 6 immediate second-order right neighbour is treatment number 5 and common second-order right neighbour series is ($1 \leq i \leq s$) i.e.1,2and 3.

lastly, for treatment number 7,8&9 the common second- order right neighbours are 4,5&6. As the treatment number 7,8&9 lies in the series ($2s+1 \leq i \leq s^2$) so the common right second-order series of these should be ($4s+1 \leq i \leq s^2+2s$) as the circularity holds for the complete design also,($4s+1 \leq i \leq s^2+2s$) reduces to ($s+1 \leq i \leq 2s$) which are there as s+1,s+2,s+3(2s) i.e,4,5,6 . As the immediate second-order right neighbour of treatment number i should be i+2 so the other member of neighbours of treatment number 7 is 9. One more neighbour of treatment number 8 is 9 which simply can't be defined as i+2. As the property of circularity holds for each set of s treatments so treatment number 8 has immediate second-order right neighbour as treatment number 7. Similarly, for treatment number 9 immediate second-order right neighbour is treatment number 8 because of the property of circularity.

2.2 Neighbour Design Using OS1 series when s=4.

The resultant neighbour design for OS1 series using MOLS for s=4 with parameters v=16,b=20,r=5,k=4,λ=1 is as:

4	8	12	16	13	14	15	16	16	15	14	13	15	14	16	13	14	16	15	13
1	5	9	13	1	2	3	4	1	2	3	4	1	4	2	3	1	3	4	2
2	6	10	14	5	6	7	8	6	5	8	7	8	5	7	6	7	5	6	8
3	7	11	15	9	10	11	12	11	12	9	10	10	11	9	12	12	10	9	11
4	8	12	16	13	14	15	16	16	15	14	13	15	14	16	13	14	16	15	13
1	5	9	13	1	2	3	4	1	2	3	4	1	4	2	3	1	3	4	2

For OS1 series when s=4 the second- order left and right neighbour cannot be find out simultaneously (for getting second- order two sided neighbour treatment simultaneously there must be at least $s \geq 5$.i.e.the block size $k \geq 5$) that is only one directional second- order neighbour can be obtained for s=4. In the above design in block 1 treatment number 3 is the second- order right neighbour of treatment number 1. Similarly, all the second- order right neighbours can be obtained for treatment number 1 in which block the treatment number 1 appears. Thus a list of second-order left neighbours for treatment number 1 so obtained is 3,9,11,10&12. Further in block 1 treatment 4 is the second- order right neighbour of treatment number 2. In block 6 treatment number 10 is the second- order right neighbour of treatment number 2. Similarly all other second- order right neighbours for treatment number 2 can be find out. Thus a list of second-order right neighbours for treatment number 2 as 4,10,12,9&11. Similarly, all the second-order neighbours for every other treatments can also be obtained and the resulted neighbours are given in Table 3 which is as follows:

Table 3

i	Second- order right Neighbours	Common neighbours	Other neighbour	Series
1	3,9,11,10,12	9,10,11,12	3	$1 \le i \le s$ i.e. $1 \le i \le 4$
2	4,10,12,9,11	9,10,11,12	4	
3	1,11,9,12,10	9,10,11,12	1	
4	9,11,10,12,2	9,10,11,12	2	
5	7,13,15,14,16	13.14.15.16	7	$s+1 \le i \le 2s$ i.e. $5 \le i \le 8$
6	8,14,16,13,15	13,14,15,16	8	
7	5,15,13,16,14	13,14,15,16	5	
8	6,16,14,15,13	13,14,15,16	6	
9	11,1,3,2,4	1,2,3,4	11	$2s+1 \le i \le 3s$ i.e. $9 \le i \le 12$
10	12,2,4,1,3	1,2,3,4	12	
11	2,4,1,3,9	1,2,3,4	9	
12	10,4,2,3,1	1,2,3,4	10	
13	8,6,7,5,15	5,6,7,8	15	$3s+1 \le i \le s^2$ i.e. $13 \le i \le 16$
14	16,6,8,7,5	5,6,7,8	16	
15	13,7,5,8,6	5,6,7,8	13	
16	14,8,6,7,5	5,6,7,8	14	

From the Table 3 it is observed that treatment number 1 has neighbours 9,10,11,12& 3. As the treatment number 1 lies in the series ($1 \le i \le s$) so the second-order right neighbour series of it shall be ($2s+1 \le i \le 3s$) which are there as 2s+1,2s+2,2s+3,2s+4(3s). As the immediate second-order right neighbour of treatment number i=1 should be i+2 so the other member of neighbours is 3. Now, the treatment number 2 has 4,10,12,9&11 as second-order right neighbours. As the treatment number 2 lies in series ($1 \le i \le s$) which has ($2s+1 \le i \le 3s$) as the series of second-order right neighbours. Immediate second-order right neighbour of treatment number 2 is 4 which simply can be written as i+2. For treatment number 3 the second-order common right neighbours are 9,10,11and12 and one more neighbour treatment is 1 which simply can't be defined as i+2. In case of OS1 series, v=s2 where s is a prime number or power of a prime number circularity not only holds for the complete design but it also holds for each set of s treatments. Hence for the treatment number 3 immediate second-order right neighbour is treatment number 1. Similarly, for treatment number 4 immediate second-order right neighbour is treatment number 2 and common second-order neighbours are again 9,10,11and12.

Secondly, it is observed that treatment number 5 has second-order right neighbours as 13,14,15,16 and 7. As the treatment number 5 lies in the series ($s+1 \le i \le 2s$) so the second –order right neighbour series of it shall be ($3s+1 \le i \le s2$) which are there as 3s+1,3s+2,3s+3,3s+4(s2). As the immediate second-order right neighbour of treatment number i should be i+2 so the other member of treatment number 5 is 7.Further we observe for the treatment number 6 which has 13,14,15,16&8 as second-order right neighbours. As the treatment number 6 lies in the series (s+1≤i≤2s) which has (3s+1≤i≤s2)as the series of second-order common right neighbours. Immediate second-order right neighbour of treatment number 6 is 8 which simply can be written as i+2 immediate second-order neighbour. Treatment number 7 has the common second –order right neighbour series as 13,14,15 and 16. One more neighbour of treatment number 7 is 5 which again can't be defined as i+2. As circularity not only holds for the complete design but it also holds for each set of s treatments. Hence for the treatment number 7 immediate second-order right neighbour is treatment number 5. Similarly, for treatment number 8 immediate second-order right neighbour is treatment number 6 and common second-order right neighbour series is (3s+1≤ i≤s2) i.e.13,14,15 and 16.

Similarly, for treatment number 9,10,11 and 12 has the common second- order right neighbours as 1,2,3,4. As the treatment number 9,10,11 and 12 lies in the series (2s+1≤i≤3s) so the second-order right neighbour series of these should be (4s+1≤i≤5s) as the circularity holds for the complete design ,(4s+1≤i≤5s) reduces to 1≤i≤s which are there as 1,…,4(s). As the immediate second-order right neighbour of treatment number i should be i+2 so the other member of neighbours of treatment number 9 is 11 and for the treatment number 10 is 12. One more neighbour of treatment number 11 is 9 which simply can't be defined as i+2. As the property of circularity holds for each set of s treatments so treatment number 10 has immediate second-order right neighbour as treatment number 9. Similarly for treatment number 12 immediate second-order right neighbour is treatment number 10 because of the property of circularity.

Lastly, for treatment number 13,14,15 and 16 has the common second –order right neighbours as 5,6,7and 8. As the treatment number 13,14,15 and 16 lies in the series (3s+1≤ i≤s2) so the second-order right neighbour series of these treatments should be (s+1≤i≤2s) which are there as s+1,s+2,s+3,s+4(2s)because of the circularity of the complete design. One more neighbour of treatment number 13 is treatment number 15 which can be defined as i+2 immediate right second-order neighbour. Treatment number i=14 has treatment number i+2=16 as immediate second-order right neighbour. As we said before for OS1 Series circularity holds for each set of s treatments therefore treatment number 15 has immediate second-order right neighbour is treatment number 13 and similarly treatment number 14 is the immediate second –order right neighbour of treatment number 16.

Now, the second-order left neighbours can be obtained from the resulted neighbour design, given above for s=4. In block 1 treatment number 3 is the second-order left neighhbour of treatment number 1. Similarly one can find out all the second-order left neighbours can be obtained in which block treatment number 1 appears. By doing so a list of second-order left neighbours for treatment number 1 is 3,9,11,10&12. In block 1 treatment number 4 is the second-order left neighbour of treatment number 2. In block 6 treatment number 10 is the second- order left neighbour of treatment number 2. Thus a list of second-order left neighbours for treatment number 2 is 4,10,12,9&11. The second-order left neighbours for every other treatment can be obtained.

After obtaining all the second-order left neighbour treatments it is found that for every treatment second-order left neighbours and second-order right neighbours are same. From the picturisation it is observed that treatment number 1 has i-2 immediate second-order left neighbour as 9 and the i+2 immediate second-order right neighbour is also 9. The second-order left neighbours and second-order right neighbours occur at the same position because of the circularity of neighbour treatments. So the common series of second-order left neighbours is also the common series of second-order right neighbours of each treatment but this result is true only for s=4.

2.3 Neighbour Designs using OS1 series when s=5.

The resultant neighbour design for OS1 series when s=5. The parameters are v=25, b=30,r=6,k=5,λ=1

5	10	15	20	25	21	22	23	24	25	22	23	24	25	21
1	6	11	16	21	1	2	3	4	5	1	2	3	4	5
2	7	12	17	22	6	7	8	9	10	10	6	7	8	9
3	8	13	18	23	11	12	13	14	15	14	15	11	12	13
4	9	14	19	24	16	17	18	19	20	18	19	20	16	17
5	10	15	20	25	21	22	23	24	25	22	23	24	25	21
1	6	11	16	21	1	2	3	4	5	1	2	3	4	5

And

24	22	25	23	21	23	25	22	24	21	25	24	23	22	21
1	4	2	5	3	1	3	5	2	4	1	5	4	3	2
8	6	9	7	10	9	6	8	10	7	7	6	10	9	8
15	13	11	14	12	12	14	11	13	15	13	12	11	15	14
17	20	18	16	19	20	17	19	16	18	19	18	17	16	20
24	22	25	23	21	23	25	22	24	21	25	24	23	22	21
1	4	2	5	3	1	3	5	2	4	1	5	4	3	2

From the above design two sided (left and right) neighbours simultaneously can be obtained. In block 1 treatment number 4 is the second- order left neighbour and treatment number 3 is the second- order right neighbour of treatment number 1. In block 11 treatment 18 is the second- order left neighbour and treatment 14 is the second order right neighbour of treatment number 1. Similarly, all two- sided neighbours for treatment number 1 in which block this treatment appears can be obtained. Thus a list of two-sided second-order neighbours of treatment number 1 is 4,3,16,11,18,14,17,15,20,12,19,13. Now the treatment number 2 has treatment 5 is the second -order left neighbour and treatment 4 is the second - order right neighbour in block 1. In block 7 treatment number 17 is the second- order left neighbour and treatment number 12 is the second- order right neighbour of treatment number 2. Similarly all the two-sided second-order neighbours can be find out easily for treatment number 2 in which block this treatment number appears. By doing so a list of second -order neighbours as 5,4,17,12,19,15,18,11,16,13,20,14. Similarly the second- order nearest neighbours for all other treatments can be find out which are given in Table 4 :

Table 4

i	Two sided Neighbours	Left neighbours	Right neighbours	Other Neighbours	Series
1	4,3,16,11,18,14, 17,15,20,12,19,13	16,17,18,19,20	11,12,13,14,15	4,3	1≤i≤s i.e. 1≤i≤5
2	5,4,17,12,19,15, 18,11,16,13,20,14	16,17,18,19,20	11,12,13,14,15	5,4	
3	1,5,18,13,20,11, 19,12,17,14,16,15	16,17,18,19,20	11,12,13,14,15	1,5	
4	2,1,19,14,16,12, 20,13,18,15,17,11	16,17,18,19,20	11,12,13,14,15	2,1	
5	3,2,20,15,17,13, 16,14,19,11,18,12	16,17,18,19,20	11,12,13,14,15	3,2	
6	9,8,21,16,23,19, 22,20,25,17,24,18	21,22,23,24,25	16,17,18,19,20	9,8	s+1≤i≤2s i.e. 6≤i≤10
7	10,9,22,17,24,20, 23,16,21,18,25,19	21,22,23,24,25	16,17,18,19,20	10,9	
8	6,10,23,18,25,16, 24,17,22,19,21,20	21,22,23,24,25	16,17,18,19,20	6,10	
9	7,6,24,19,21,17, 25,18,23,20,22,16	21,22,23,24,25	16,17,18,19,20	7,6	
10	8,7,25,20,22,18, 21,19,24,16,23,17	21,22,23,24,25	16,17,18,19,20	8,7	
11	14,13,1,21,3,24, 2,25,5,22,4,23	1,2,3,4,5,	21,22,23,24,25	14,13	2s+1≤i≤3s i.e 11≤i≤15
12	15,14,2,22,4,25, 3,21,1,23,5,24	1,2,3,4,5	21,22,23,24,25	15,14	
13	11,15,3,23,5,21, 4,22,2,24,1,25	1,2,3,4,5	21,22,23,24,25	11,15	
14	12,11,4,24,1,22,5 ,23,3,25,2,21	1,2,3,4,5	21,22,23,24,25	12,11	
15	13,12,5,225,2,23, 1,24,4,21,3,22	1,2,3,4,5	21,22,23,24,25	13,12	
16	19,18,6,1,8,4,10, 5,9,2,7,3	6,7,8,9,10	1,2,3,4,5	19,18	3s+1≤i≤4s i.e. 16≤i≤20
17	20,19,7,2,9,5,8, 1,6,3,10,4	6,7,8,910	1,2,3,4,5	20,19	
18	16,20,8,3,10,1,9, 2,7,4,6,5	6,7,8,9,10	1,2,3,4,5	16,20	
19	17,16,9,4,6,2, 10,3,8,5,7,1	6,7,8,9,10	1,2,3,4,5	17,16	
20	18,17,7,5,6,3, 9,4,8,1,10,2	6,7,8,9,10	1,2,3,4,5	18,17	
21	24,23,11,6,13, 9,12,10,15,7,14,8	11,12,13,14,15	6,7,8,9,10	24,23	$4s+1 \leq i \leq s^2$ i.e. 21≤i≤25
22	25,24,12,7,14, 10,13,6,11,8,15,9	11,12,13,14,15	6,7,8,9,10	25,24	
23	21,25,13,8,15,6, 14,7,12,9,11,10	11,12,13,14,15	6,7,8,9,10	21,25	
24	22,21,14,9,11,7, 15,8,13,10,12,6	11,12,13,14,15	6,7,8,9,10	22,21	
25	23,22,15,10,12,8, 11,9,14,6,13,7	11,12,13,14,15	6,7,8,9,10	23,22	

From table number 4 it is observed that treatment number 1,2,3,4 & 5has neighbours 16,17,18,19 and 20 as common second-order left neighbours and treatment number 11,12,13,14&15 as the common second-order right neighbours. The treatment number 1,2,3,4& 5 lies in the series (1≤i≤s) so the common second –order left neighbour series of these should be (3s+1≤ i≤ 4s) which are there as 3s+1,3s+2,3s+3,3s+4,3s+5(4s) i.e.16,17,18,19&20. The treatment number 1,2,3,4and 5 lies in the series (1≤i≤s)so the common second-order right neighbour series of these treatments should be (2s+1≤i≤3s) which are there as 2s+1,2s+2,2s+3,2s+4(3s) i.e.11,12,13,14&15. Two more neighbours of treatment number 1 are 4 and 3. As the concept of second-order means these are immediate i-2 left second order and i+2 immediate right second- order neighbour respectively. Two more neighbour treatments of treatment number 1 are 4 and 3. Treatment number 4 can't be simply written as i-2 as the immediate second-order left neighbour. In case of OS1 series, circularity not only holds for the complete design but it also holds for each set of s treatments. Hence for treatment number 1 immediate second- order left neighbour is treatment number 4. Treatment number 3 is the immediate i+2 second –order right neighbour of treatment number 1.For treatment number 2 treatment number 5 is the immediate second-order left neighbour because of the property of circularity holds and treatment number 4 is the immediate second-order right neighbour. Treatment number 3 has other neighbours are 1 and 5 which are simply immediate i-2 and i+2 second –order neighbours. Treatment number 4 has two other neighbours are 2 and 1. Treatment number 2 is the immediate i-2 second-order left neighbour and treatment number 1 can be simply written as i+2 because of the property of circularity of neighbour treatments for each set of s treatments. Similarly, for treatment number 5 treatment number 3 is the immediate second-order left neighbour and treatment number 2 is the immediate second-order right neighbour.

Secondly, it is observed that treatment number 6,7,8,9&10 has neighbours 21,22,23,24&25 as common second-order left neighbours and treatment no.16,17,18,19&20 as common second-order right neighbours. The treatment number 6,7,8,9&10 lies in the series (s+1≤i≤2s) so the common second –order left neighbour series of these should be (4s+1≤i≤s2) which are there as 4s+1,4s+2,4s+3,4s+4,4s+5(s2). The treatment number 6,7,8,9&10 lies in the series (s+1≤i≤2s) so the second-order right neighbour series of these treatments should be (3s+1≤i≤4s) which can be written as 3s+1,3s+2,3s+3,3s+4,3s+5(4s). Now two more neighbours of treatment number 6 are 9 and 8. As the concept of second-order both- sided these are immediate i-2 second order left and i+2 immediate second- order right neighbour treatment. Treatment number 9 can't be simply written as i-2 as the immediate second-order left neighbour. As circularity not only holds for the complete design but it holds for each set of s treatments. Hence for treatment number 6 second- order left neighbour is treatment number 9 and treatment number 8 is the immediate second –order right neighbour of treatment number 6. For treatment number 7 treatment number 10 is the immediate i-2 second-order left neighbour because the property of circularity holds and treatment number 9 is the immediate i+2 second-order right neighbour. Treatment number 8 has other neighbours as 6 and 10 which are simply immediate i-2 second-order left and i+2 second –order right neighbours. Treatment number 9 has two other neighbours as 7 and 6. For treatment number 9 Immediate left i-2 second-order neighbour treatment is treatment number 7 and Immediate second-order right neighbour is treatment number 6 can be simply written as i+2 because of property of circularity of neighbour treatments for each set of s treatments. Similarly, for treatment no. 10 treatment no.8 is the immediate left second-order neighbour and treatment no. 7 is the immediate right second-order neighbour treatment because of the property of circularity hold for each set of s treatments.

Similarly, the treatment number 11,12...20 can be defined using property of circularity.

Lastly, it is observed that treatment number 21,22,23,24,&25 has neighbours 11,12,13,14& 15 as common second-order left neighbours and treatment number 6,7,8,9,&10 as common second-order right neighbours. The treatment number 21,22,23,24&25 lies in the series (4s+1≤i≤s2) so the common second –order left neighbour series of these should be (2s+1≤i≤3s) which are there as 2s+1,2s+2,2s+3,2s+4,2s+s(3s). The treatment number 21,22,23,24& 25 lies in the series (4s+1≤i≤s2)so the second-order right neighbour series of these treatments should be ($s+1 \leq i \leq 2s$) so the right neighbours can be written as s+1,s+2,s+3,s+4,s+5(2s) because of the property of circularity. Two more neighbours of treatment number 21 are 24 and 23. As the concept of second-order both-sided these are immediate i-2 left second order and i+2 immediate right second- order treatment. Treatment number 24 can't be simply written as i-2 as the immediate left second-order neighbour of treatment number 21. As circularity not only holds for the complete design but it holds for each set of s treatments, hence for treatment number 21 second- order left neighbour is treatment number 24 and the immediate i+2 second –order right neighbour is treatment number 23.For treatment number 22 treatment number 25 is the immediate second-order left neighbour because of the property of circularity and treatment number 24 is the immediate second-order right neighbour. Treatment number 23 has other neighbours as 21 and s25 which are simply immediate i-2 left and i+2 right second –order neighbours. Treatment number 24 has two other neighbours as 22 and 21. For treatment number 24 treatment number 22 is the immediate second-order left neighbour and treatment number 21 can be simply written as i+2 because of property of circularity . Similarly, for treatment number 25 treatment number 23 is the immediate second-order left neighbour and 22 is the second-order right neighbour treatment because of the property of circularity holds for the complete design as well as for each set of s treatments.

Conclusion:

By using the assumption that no treatment is (i) adjacent to itself and (ii) adjacent to any other treatment more than once in Neighbour Design constructed for the series $v=s2, b=s(s+1), r=s+1, k+s, \lambda=1$(s is either a prime or power of a prime number) the second –order left and right neighbours are obtained considering the property of circularity. For s=3 and s=4 the left and right second-order neighbours can't be find out simultaneously. For this the block size must be greater than 5 that is $k \geq 5$. For s=3, only one directional left and right second-order neighbours can be obtained. It is found that there exist s series in total and one common series of neighbours is there for each set of s treatments. Also for s=4 treatments using one direction can be obtained and it is observed that right second-order neighbours and left second –order neighbours for every treatment occur at the same position. For $s \geq 5$ left second-order neighbours and right second-order neighbours are obtained simultaneously. The second-order left neighbours follows the property of second-order left circularity and second-order right neighbours follows the property of second-order right circularity. It is concluded that circularity holds for the set of s treatments as well as for the common series whatever may be the size of k.

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