



ANALYSIS OF AN INHOMOGENEOUSLY LOADED HELIX ENCLOSED IN A VANE LOADED CYLINDRICAL WAVEGUIDE



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Abstract: Cylindrical waveguide loaded when helix slow-wave structure (SWS) several distinct modes arises. There would include the fast-wave modes corresponding to the hybrid of the transverse electric (TE) and transverse magnetic (TM) modes arising from the anisotropic conductivity of the helix which internally loads the overall metal envelope — a conventional cylindrical waveguide. Analysis of helical structure in the first-wave regime with arbitrary azimuthal harmonic modes has been carried out to ensure wideband coalescence between the beam-mode and the waveguide-mode dispersion curves. In this paper, authors have developed an analysis of the structure considering the effects of the dielectric-support parameters including the inhomogeneity of the dielectric in a vane loaded cylindrical waveguide.

Keyword: Helix slow-wave structure, Fast-wave device, Gyro-TWT.

INTRODUCTION :

The helix, a non-resonant electromagnetic structure, finds several applications and as a SWS in a traveling-wave tube (TWT) and hence the analysis of the helical structure is carried out in the slow-wave regime and reported elsewhere [1]-[5]. However, there could be several modes when a vane loaded cylindrical waveguide is loaded with helix SWS [6]-[12] and, thus, can find potential application in gyro-TWT.

These modes would include the fast-wave modes corresponding to the hybrid of the TE and the TM modes arising from the anisotropic conductivity of the helix which internally loads the anisotropic overall metal envelope appeared as a corrugated cylindrical waveguide. Helix SWS has been analyzed in the fast-wave regime with arbitrary azimuthal harmonic modes, keeping in mind its potential application in gyro-resonance electron beam devices, like, the gyro-TWT a high-power, millimeter-wave amplifier for communication systems [5]-[12]. The objective is to control the structure parameters with a view to shaping its dispersion (-) characteristics which would consequently ensure a wideband coalescence between the beam-mode and the waveguide-mode dispersion curves. In analyzing the structure in the fast-wave regime the effects of the dielectric-support system for the helix is ignored [7]-[12], but considered in [6] for an isotropic corrugated cylindrical waveguide. However, in these analyses, anisotropic conductivity of cylindrical waveguide, realized by providing metallic vanes radially inward from the waveguide, is not included. Here, the analysis has been done of the structure considering the effects of the dielectric-support parameters including the inhomogeneity of the dielectric in vane loaded cylindrical waveguide. The position of metal vanes in addition to other structure parameters, namely, structure inhomogeneity, helix pitch angle, etc., controls dispersion characteristics for wideband coalescence between the beam-mode and the waveguide-mode dispersion curves. Position of metal vanes with respect to the helix, control the dispersion mode and gives a flat dispersion which is eventually gives wide coalescence between beam-mode and waveguide-mode.

The structure consists of a helix, inhomogeneously loaded identical dielectric tube regions [4]-[6] and vane loaded waveguide (Fig. 1). Identical dielectric tube regions are obtained by azimuthally smoothing the dielectric tube regions to an effective permittivity value [2], [4]-[6]. In the present structure, metal vanes are internally loaded with an inhomogeneously loaded helix and modeled analytically. Number of dielectric layers increased till results are converged.

2. ANALYSIS

2.1. Field expressions

Due to the anisotropic conductivity of the sheath-helix, one expects the excitation of both TE and TM modes in the structure. The relevant components of electric (Ez) and magnetic (Hz) field intensities are [6]-[12]:

$$E_{z,p} = \sum_{m=-\infty}^{\infty} E_{z,m,p} = \sum_{m=-\infty}^{\infty} (A_{m,p} J_m \{\gamma_{m,p} r\} + B_{m,p} Y_m \{\gamma_{m,p} r\}) \quad (1)$$

$$H_{z,p} = \sum_{m=-\infty}^{\infty} H_{z,m,p} = \sum_{m=-\infty}^{\infty} (C_{m,p} J_m \{\gamma_{m,p} r\} + D_{m,p} Y_m \{\gamma_{m,p} r\}) \quad (2)$$

$$E_{\theta,p} = \sum_{m=-\infty}^{\infty} E_{\theta,m,p} = \sum_{m=-\infty}^{\infty} \left[\frac{j\beta_m}{r\gamma^2} \frac{\partial}{\partial \theta} E_z + \frac{\mu_0}{\gamma^2} \frac{\partial}{\partial t} \frac{\partial}{\partial r} H_z \right] \quad (3)$$

$$H_{\theta,p} = \sum_{m=-\infty}^{\infty} E_{\theta,m,p} = \sum_{m=-\infty}^{\infty} \left[-\frac{\varepsilon_0 \varepsilon'_r}{\gamma^2} \frac{\partial}{\partial t} \frac{\partial}{\partial r} E_z + \frac{j\beta_m}{r\gamma^2} \frac{\partial}{\partial \theta} H_z \right] \quad (4)$$

$$E_{r,p} = \sum_{m=-\infty}^{\infty} E_{\theta,m,p} = \sum_{m=-\infty}^{\infty} \left[-\frac{1}{\gamma^2} \frac{\partial}{\partial z} \frac{\partial}{\partial r} E_z + \frac{\mu_0}{r\gamma^2} \frac{\partial}{\partial t} \frac{\partial}{\partial \theta} H_z \right] \quad (5)$$

$$H_{r,p} = \sum_{m=-\infty}^{\infty} E_{\theta,m,p} = \sum_{m=-\infty}^{\infty} \left[-\frac{\varepsilon_0 \varepsilon'_r}{r\gamma^2} \frac{\partial}{\partial t} \frac{\partial}{\partial \theta} E_z + \frac{1}{\gamma^2} \frac{\partial}{\partial z} \frac{\partial}{\partial r} H_z \right] \quad (6)$$

Assuming rf fields to vary as $\exp j(\omega t - \beta z - m\theta)$ where $\beta_m (= \beta_0 + m \cot \psi)$ is the axial phase propagation constant of m^{th} space harmonics $\gamma_m (= (\beta_m^2 - \varepsilon'_{r,\text{eff}} k^2)^{0.5})$ is the radial propagation constant, a is the mean helix radius and $k (= \omega / c = \omega (\mu_0 \varepsilon_0)^{0.5})$ is the free-space propagation constant. P refers to a region of the structure. $J_m \{\gamma_m r\}$ and $K_m \{\gamma_m r\}$ are the m^{th} order Bessel's functions of the first and second kinds and their prime indicating their derivatives with respect to argument, respectively. $A_p, B_p,$ and D_p are the field constants.

2.2. Dispersion characteristics

The analytical model (Fig. 1) consists of two free-space regions, i) inside the helix and ii) between helix mean radius to outer helix radius, iii) 'n' number of dielectric tube regions and iv) an inter vane region.

The relevant electromagnetic boundary conditions for the problem are: the tape-helix boundary conditions [6] at $r = a$, the boundary conditions related to the continuity of the tangential components of the electric and magnetic field intensities at each of the interfaces ($r = bp$) between the dielectric tube regions (between the p th and $(p+1)$ th regions; $2 \leq p \leq n+1$) as well as those between the free-space region ($p = 2$) and the first dielectric tube region ($p = 3$), and the boundary condition that the tangential components of electric field intensity are null at the vane tip ($r = bn-1 = \text{rin}$), and azimuthal components are null at cylindrical waveguide ($r = bn = \text{rwing}$). With the help of these boundary conditions and following approach [6], one may get the dispersion relation as :

$$\sum_{m=-\infty}^{+\infty} \left[(\gamma_m a + m\beta_m \cot \psi)^2 D_{LF}^2 + J'_m \{\gamma_m a\} Y'_m \{\gamma_m a\} (k \cot \psi)^2 \right] J_{//,m} = 0 \quad (7)$$

$$J_{//,m} = \frac{\sin(\beta_m \delta / 2)}{(\beta_m \delta / 2)} \quad (8)$$

With,

$$D_{LF}^2 = \frac{1 + \Re_0 J_m \{\gamma_m a\} / Y_m \{\gamma_m a\}}{1 + \Im_0 J'_m \{\gamma_m a\} / Y'_m \{\gamma_m a\}} \quad (9)$$

$$2 \leq p \leq n$$

$$\mathfrak{R}_{p-2} = \frac{Y_m \{\gamma_m b_{p-2}\} - \mathfrak{S}_{p-2} \varepsilon'_{r,p+1} Y_m \{\gamma_m b_{p-2}\}}{J_m \{\gamma_m b_{p-2}\} - \mathfrak{S}_{p-2} \varepsilon'_{r,p+1} J_m \{\gamma_m b_{p-2}\}} \quad (10)$$

$$\mathfrak{S}_{p-2} = \frac{\mathfrak{S}_{p-1} J'_m \{\gamma_m b_{p-2}\} - Y_m \{\gamma_m b_{p-2}\}}{\mathfrak{S}_{p-1} J_m \{\gamma_m b_{p-2}\} + Y_m \{\gamma_m b_{p-2}\}} \quad (11)$$

and

$$\mathfrak{R}_{n-1} = \frac{\varepsilon'_{r,n+1} Y'_m \{\gamma_m b_{n-1}\} - \mathfrak{S}_{n-1} \varepsilon'_{r,n+2} Y_m \{\gamma_m b_{p-2}\}}{J_m \{\gamma_m b_{p-2}\} - \mathfrak{S}_{p-2} \varepsilon'_{r,p+1} J_m \{\gamma_m b_{p-2}\}} \quad (12)$$

$$\mathfrak{S}_{n-1} = \frac{J'_m \{\gamma_m b_{n-1}\} Y_m \{\gamma_m b_{n-1}\} - Y'_m \{\gamma_m b_{n-1}\} J_m \{\gamma_m b_{n-1}\}}{J_m \{\gamma_m b_{n-1}\} Y_m \{\gamma_m b_{n-1}\} - Y_m \{\gamma_m b_{n-1}\} J_m \{\gamma_m b_{n-1}\}} \quad (13)$$

$$2 \leq p \leq n+1$$

$$\mathfrak{N}_{p-2} = \frac{Y_m \{\gamma_m b_{p-2}\} - \ell_{p-2} Y'_m \{\gamma_m b_{p-2}\}}{J_m \{\gamma_m b_{p-2}\} - \ell_{p-2} J'_m \{\gamma_m b_{p-2}\}} \quad (14)$$

$$\ell_{p-2} = \frac{\mathfrak{N}_{p-1} J_m \{\gamma_m b_{p-2}\} + Y_m \{\gamma_m b_{p-2}\}}{\mathfrak{N}_{p-1} J'_m \{\gamma_m b_{p-2}\} + Y'_m \{\gamma_m b_{p-2}\}} \quad (15)$$

With

$$\mathfrak{N}_{n-1} = -\frac{Y_m \{\gamma_m b_{n-1}\} - \ell_{n-1} Y'_m \{\gamma_m b_{n-1}\}}{J_m \{\gamma_m b_{n-1}\} - \ell_{n-1} J'_m \{\gamma_m b_{n-1}\}} \quad (16)$$

$$\ell_{n-1} = \frac{J_m \{\gamma_m b_{n-1}\} Y'_m \{\gamma_m r_w\} - Y_m \{\gamma_m b_{n-1}\} J'_m \{\gamma_m r_w\}}{J'_m \{\gamma_m b_{n-1}\} Y'_m \{\gamma_m r_w\} - Y'_m \{\gamma_m b_{n-1}\} J'_m \{\gamma_m r_w\}} \quad (17)$$

Where \mathfrak{R}_0 , \mathfrak{S}_0 , and ℓ_0 are functions of structure parameters.

2.3. Interaction impedance

The quality of the structure, for practical application, can be estimated in terms of the interaction impedance of the structure. However, for fast-wave analysis, considering the potential use of the structure in a fast-wave gyro TWT, that it is now predominately the azimuthal instead of the axial electric field intensity, unlike in a conventional (slow-wave) TWT, which would be involved in the beam-wave interaction of the device. Therefore, K_m , the azimuthal interaction impedance of the m th space-harmonic mode, presented here as [6] as:

$$K_{\theta,m} = \frac{E_{\theta,m}^2 \{r_{beam}\}}{2\beta_m^2 P} \quad (18)$$

where $E_m \{r_{beam}\}$ is the azimuthal component of electric field intensity of the m th space-harmonic mode at the mean beam position ($r=r_{beam}$) somewhere between the waveguide axis and the helix. P is the power propagating down the structure, which can be found by taking half of the real part of the integration of the complex Poynting vector over the cross-sectional area of the structure for all the space harmonic modes summed up together and finally appeared as a function of structure parameters

(10)-(17).

3. RESULTS AND DISCUSSION

In this paper, authors have presented an analysis, of an inhomogeneously helix-loaded structure enclosed in a vane loaded waveguide, in the fast-wave regime. The tape-helix model has been used which takes into account the effect of the space harmonics, as it is particularly relevant to situations in which the structure is operated at high voltages and for high helix pitch angles. In the analytical model, the structure inhomogeneity has been modelled by azimuthally smoothing out the discrete supports into a number of continuous dielectric tube regions of appropriate permittivity values. The number of such simulated continuous homogeneous dielectric tube regions is increased till results are converged. The effect of helix tape thickness is taken into account to add to the practical relevance of the problem. The nonuniformity of the radial propagation constant in the different structure regions is considered, particularly for a large structure inhomogeneity caused by a high-permittivity support for the helix, a smaller separation between the helix and the vane tip, a large separation between the helix and the waveguide wall, and for large helix pitch angles. Thus, the fast-wave analysis of the structure which is carried out includes quite a number of points of practical relevance.

For the numerical appreciation of the problem the dispersion is plotted for an inhomogeneously loaded helix taking r_{inv}/a , the relative position of the vane tip (Fig. 2(a)), r_{wg}/a , the relative position of the cylindrical waveguide (Fig. 2(b)),

inhomogeneity, $x (= \epsilon'_{r,p} / \epsilon'_{r,p-1} \neq 1; 3 \leq p \leq n+2)$ the relative permittivity of the dielectric tube regions (Fig. 2(c)), $\epsilon'_{r,3} (= \epsilon'_{r,p}; 3 \leq p \leq n+2)$, the relative permittivity of the first homogeneous support region (Fig. 2(d)) and \cot , the cotangent of the helix pitch angle (Fig. 2(e)) as the parameters. For this purpose, the lowest-order solution of the hybrid-mode relation (7) is taken.

It is cleared that with the proximity of vane tip (r_{inv}) to helix, loading increases and the value of β increased and a flat dispersion is achieved (Fig. 2(a)). The relative position of waveguide wall (r_{wg}) also controls the dispersion, proximity of the waveguide wall increases the loading and hence higher value of β obtained (Fig. 2(b)). For an inhomogeneously loaded structure, the loading increases with the inhomogeneity, i.e., if the relative permittivity of the support region is increased radially outward (> 1) (Fig. 2(c)). The structure loading also increases with the increase in relative permittivity of the first

dielectric tube region $\epsilon'_{r,3} (= \epsilon'_{r,p}; 3 \leq p \leq n+2)$ (Fig. 2(d)). Helix pitch angle also plays an important role in controlling the dispersion (Fig. 2(e)). With the increase in helix pitch angle loading decreases and this is a critical parameter for high frequency application, specially, when the device is operating at high operating beam voltage.

Position of the waveguide wall and the vane tip has a significant role in controlling the dispersion (Figs 2(a) and 2(b)). Close proximity of the waveguide wall to the helix gives flat dispersion characteristics. However, without bringing waveguide wall into the close vicinity of helix, position and height of vanes controls dispersion. Thus, position of the waveguide is controlled by the radial height of the vanes. If the waveguide is kept away from the helix, reasonable height of dielectric supports to the helix can be made. Thus, a vane loaded waveguide find its potential is designing a helix loaded wave guide for gyro TWTs. For a given position of the waveguide and vane tip, with the increase in inhomogeneity factor, x (Fig. 2(c)), the dispersion became flatter. With the increase in permittivity value of the first dielectric tube region dispersion also became flatter (Fig. 2(d)). There is an optimum value for both inhomogeneity factor and permittivity value of first region, such that dielectric constant at the outer most region should not exceed beyond practical limit. Helix pitch is a critical parameter for beam-wave interaction (Fig. 2(e)). Lower the helix pitch angle, flatter the dispersion. However, for vane loaded waveguide, effect of helix pitch angle is less.

It is also important to study the fundamental mode ($m = 0$), azimuthal interaction impedance ($K_{\theta,0}$), since it has direct relevance to the gain to the fast-wave device, like, gyro-TWT. Interaction impedance of the structure decreases with frequency (Fig. 3). Position of vane tip has a greater effect on interaction impedance. With the proximity of vane tip to the helix, the interaction impedance increases. Proximity of the vane is enhances cutoff frequency and there is an optimum position of vane to achieve maximum impedance (Fig. 3(a)). The proximity of the waveguide wall from the helix increases both impedance and cut-off frequency (Fig. 3(b)). With the increase in inhomogeneity factor, x , interaction impedance increases. The value of inhomogeneity factor, x , lower than unity, that is, with the decrease in dielectric loading radially outward, impedance decreases and impedance increases with the increase in x from unity (Fig. 3(c)). However, with the increase in dielectric loading of the first dielectric tube region, operating frequency decreases (Fig. 3(d)). Effect of helix pitch angle is more on interaction impedance (Fig. 3(e)). Lower the helix pitch angle, higher the interaction impedance.

Thus, there is an optimum position of vane with respect to the helix to control dispersion of the structure and to get flatter dispersion for wideband coalescence between beam-mode and waveguide-mode. Other structure parameters have also an important role in controlling the dispersion and impedance. However, variation of structure parameter for getting flatter dispersion do not reduces interaction impedance of the structure. Thus the position of the vane in addition with other structure parameters controls dispersion and finds its suitable potential application for gyro TWTs.

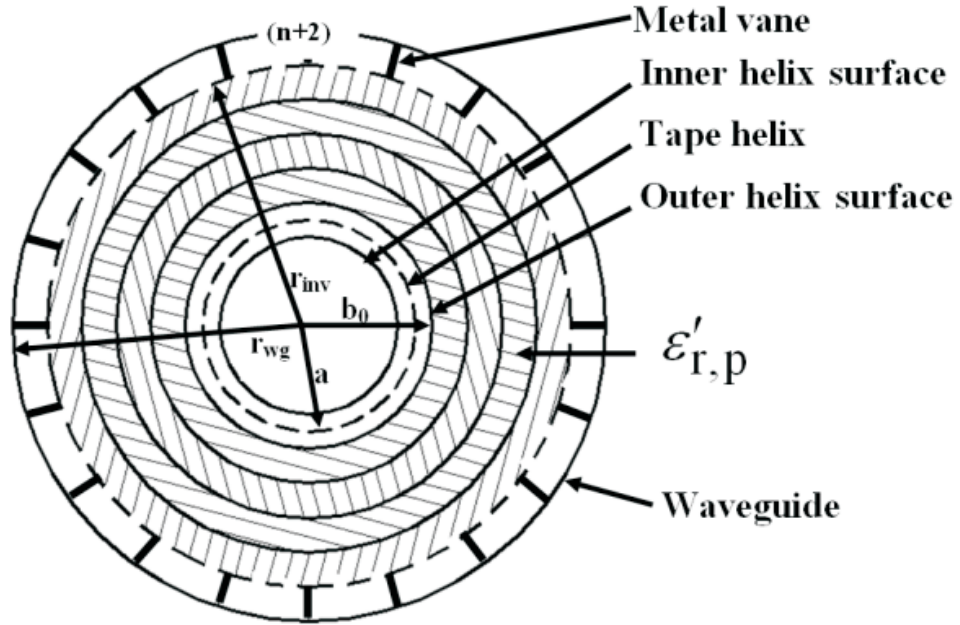


Figure 1. Analytical model vane loaded cylindrical waveguide loaded with helix and dielectric support system (dielectric supports are azimuthally smoothed out into a number of dielectric tube regions).

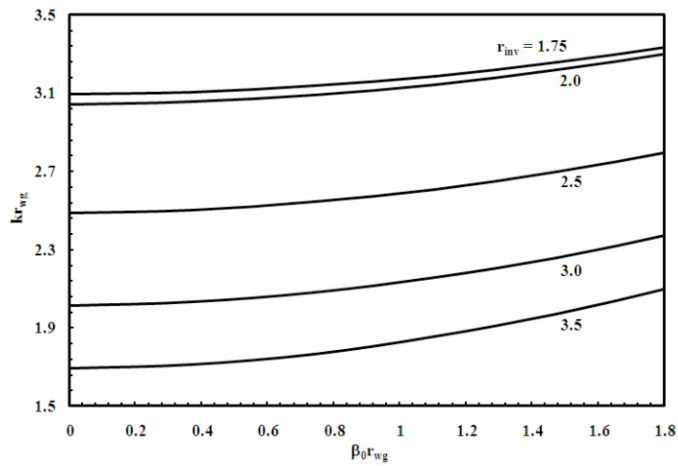


Figure 2(a)

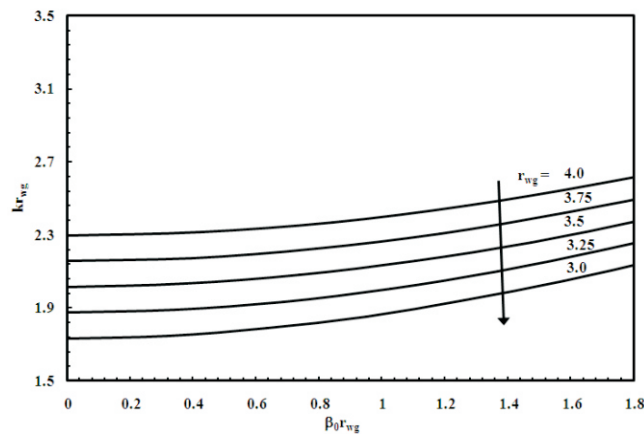


Figure 2(b)

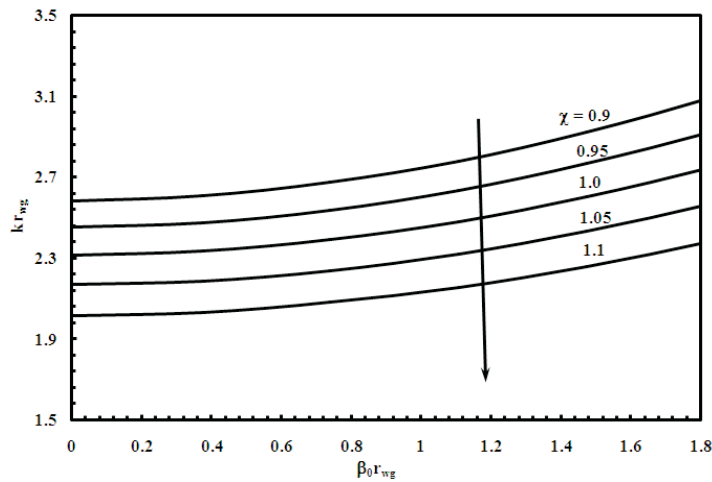


Figure 2(c)

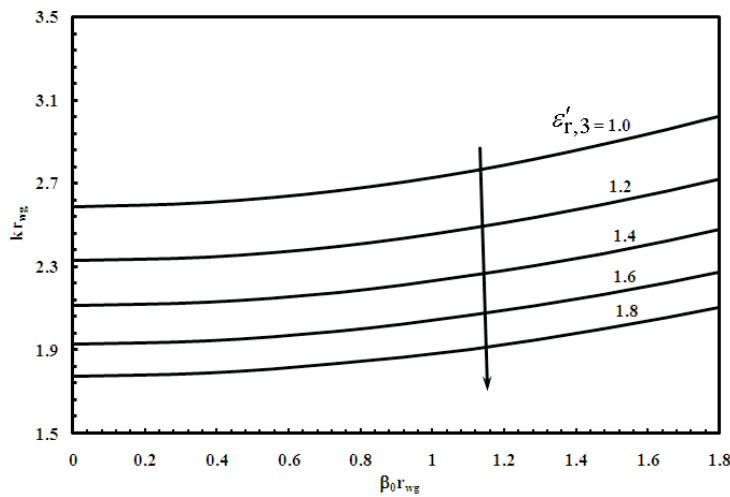


Figure 2(d)

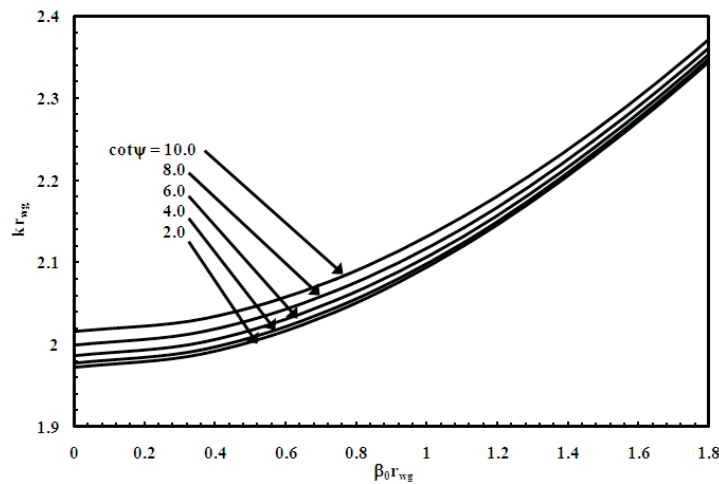


Figure 2(e)

Figure 2. Dispersion characteristics normalized frequency (kr_{wg}) versus a normalized phase propagation constant ($\beta_0 r_{wg}$) of the fundamental mode ($m = 0$), taking the following parameters: (a) relative position of the vane (r_{inv}), (b) relative position of the waveguide wall (r_{wg}), (c) the inhomogeneity factor (χ), (d) dielectric loading of the first dielectric tube region ($\epsilon'_{r,3}$)

and (e) the cotangent of the helix pitch angle ($\cot \Psi$). The thickness of the tape being ignored ($b_0 = a$), though its width considered as finite at a normalized value $\delta/p' = 0.3$, $p' (= 2\pi a / \cot \psi)$ being the helix pitch.

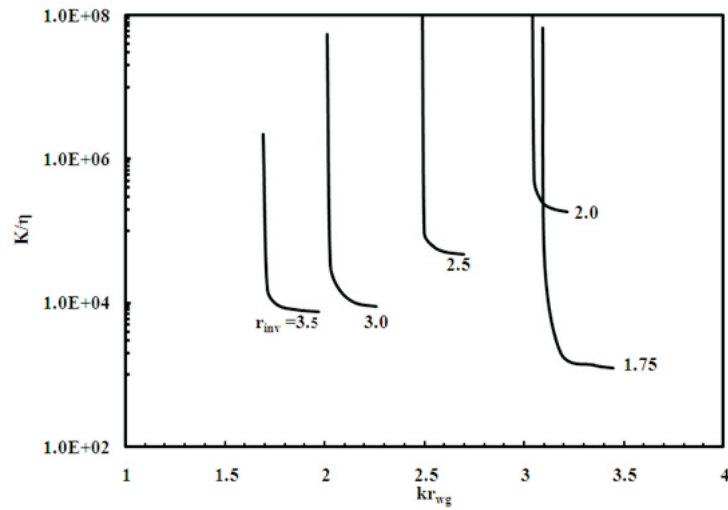


Figure 3(a)

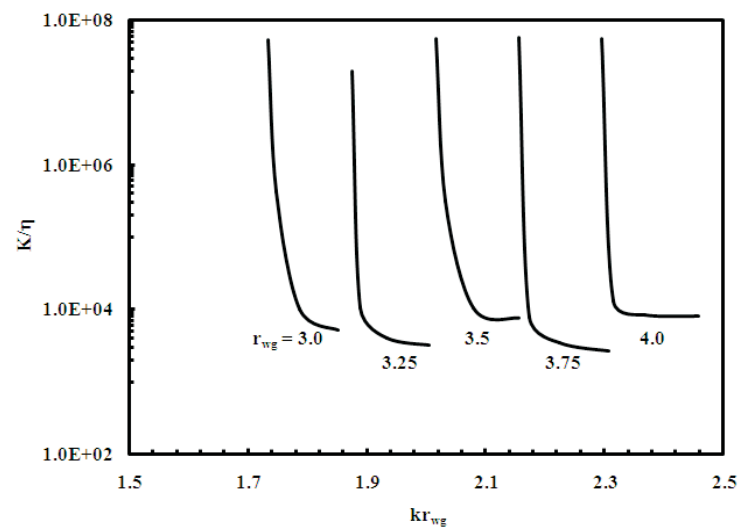


Figure 3(b)

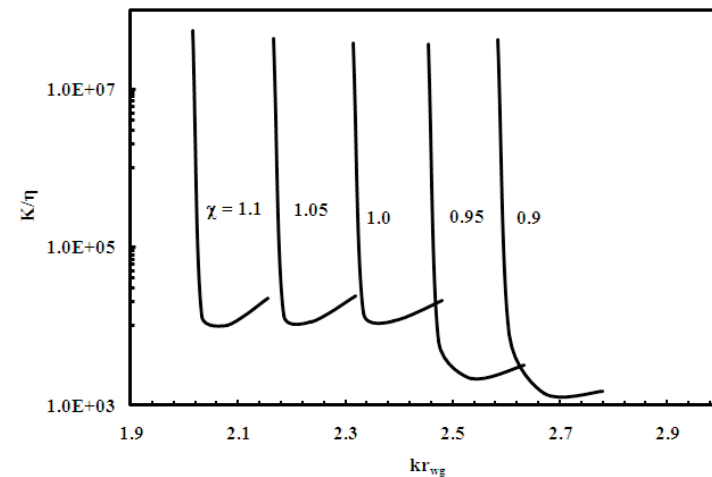


Figure 3(c)

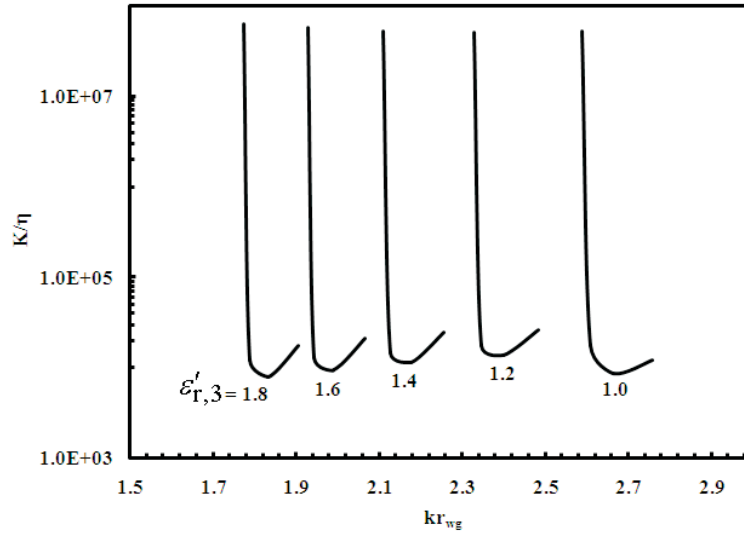


Figure 3(d)

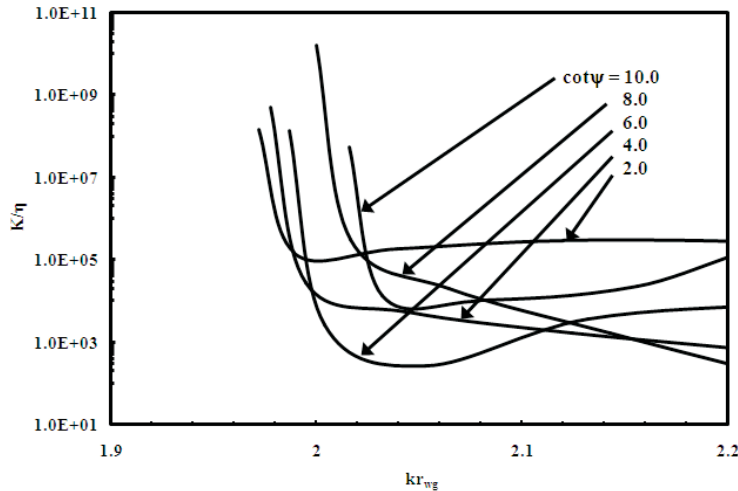


Figure 3(e)

Figure 3. The fundamental ($m = 0$) normalized interaction impedance (K/η_0) -versus-normalized frequency (kr_w) characteristics, (a)-(e) referring to the same parameters and situations as in Fig. 2, the thickness of the tape being ignored ($b_0 = a$), though its width considered as finite at a normalized value $\delta/p' = 0.3$, $p' (= 2\pi a/\cot \psi)$ being the helix pitch and the waveguide-wall radius being taken as $r_w = 1.0$ cm.

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